Cambridge **IGCSE Mathematics**

2005

Model Answers

Note the instructions ask you to give answers to 3 sig figs, where appropriate. (In general, the number of significant figures in an answer should not exceed the number of significant figures in the input data, or if this data has differing numbers of significant figures, the data with the lowest number of significant figures).

Brian Daugherty

Statements in italics are for information rather than a part of the answer

Paper 1 - 0580/01 - 0581/01, May/June 2005

Question 1

$$1393000km \quad \text{to 4 sig figs}$$

Question 2

$$\frac{9}{30} = \frac{3}{10}$$

Question 3

as a fraction

$$\frac{20}{50}$$

as a percentage

$$\frac{20}{50} \times 100 = 40\%$$

I have written the procedure in full for the benefit of those who might have had difficulty understanding the full method

Question 4

$$3.5kg : 800g$$

$$3500 : 800$$

$$35 : 8$$

Question 5

$$4^{-3} = \frac{1}{4^3} = \frac{1}{64}$$

Question 6

(a)

$$12$$

(b)

$$3$$

Note that 3 is the only <u>*prime*</u> *factor of 27*

Question 7

$$\frac{28}{4} \times 9 = \$63$$

i.e. divide by 4 to get $\frac{1}{9}$*the and then multiply by 9 to get the full amount*

Question 8

$$x^3 + 2x^2$$

$$= (-3)^3 + 2(-3)^2$$

$$= -27 + 2(9)$$

$$= -27 + 18$$

$$= -9$$

Question 9

$$255g \leq \text{weight} < 265g$$

Question 10

$$\sin 16 = \frac{h}{12}$$

$$h = 12 \sin 16$$

$$= 3.308$$

$$= 3.31 \quad \text{to 3 sig figs}$$

sines are very rarely exact, so apply instruction to state answers to 3 significant figures

Question 11

$$5000 \times \frac{6}{100} \times 3 = 900$$

could have used 0.6 instead of $\frac{6}{100}$

Question 12

$$p = st - q$$

$$st = p + q$$

$$s = \frac{p+q}{t}$$

Question 13

(a) similar

this scenario is a common example of similar triangles

(b)

$$145°$$

interior/supplementary angles

Question 14

$$\pi r^2 h = \pi(15)^2 \times 2$$

$$= 450\pi$$

$$= 1413.717mm^3$$

Question 15

(a)

$$24$$

a usual way of getting a common multiple is just to multiply the two numbers together, so could have 48 - it's just that in this case 24 is smaller and therefore probably the best one to choose, given that you then have to use this common multiple in calculations

(b)

$$\frac{5}{6} - \frac{3}{8}$$
$$= \frac{20}{24} - \frac{9}{24}$$
$$= \frac{11}{24}$$

Question 16

(a)

$$23$$

the numbers go up by 4 each time

(b) Since the numbers go up by four each time

the 10th term = 43

(c) the first differences are a constant, i.e. 4

$$\Rightarrow \text{ formula is of form } 4n + k$$

compare the series $4n$ (i.e. 4,8, 12, 16,...) with the original series:-

to see that $k = 3$

so general formula is

$$4n + 3$$

Question 17

(a)

$$7x + 5 - 3(x - 4)$$
$$= 7x + 5 - 3x + 12$$
$$= 4x + 17$$

(b)

$$5x^2 - 7x = x(5x - 7)$$

Question 18

bananas

$$1.5 \times 80 = 120$$

yams

$$3 \times 45 = 135$$

so

$$\text{total} = 120 + 135 = 255, \quad i.e. \ \$2.55$$

and money left

$$= 5 - 2.55 = \$2.45$$

Question 19

(a) (i)

$$\frac{9 - 3 \times 2}{3}$$

(ii)

$$\frac{9 - 6}{3} = \frac{3}{3} = 1$$

(b)

$$1.007 = 1.01 \quad \text{to 2 d.p.}$$

Question 20

(a) Panama < Guyana < Colombia < Brazil

(b)

$$\frac{1.14 \times 10^6}{2.15 \times 10^5}$$

$$= \frac{11.4 \times 10^5}{2.15 \times 10^5}$$

$$= 5.302$$

so insert the whole number

$$5$$

Question 21

(a)

$$16 \times \frac{35}{100} = \$5.60$$

as before, could use 0.35 instead of $\frac{35}{100}$

(b) Half price is \$8 off

so he would have saved an extra \$2.40

Question 22

(a) In $\frac{1}{2}$ hour he cycled 5 km $\Rightarrow$ speed of $10km/h$

(b)

$$20 \ mins$$

(c) Draw line thru (8.00, 0) and (9,15), passing through (8.30,7.5), which will reach 12 hours at approx. 0848

(d) difference between 0848 and 0900

$$= 12 \ mins$$

Question 23

(a)

$$\angle DCE = 90^\circ$$

angle subtended by diameter at circumference

(b)

$$180 - (90 + 25)$$

$$= 65^\circ$$

(c)

$$\angle DEB = 90$$

$$\angle DBE = 180 - (90 + 65)$$
$$= 25^\circ$$

Paper 2 - 0580/02 - 0581/02, May/June 2005

Question 1

(a)

$$\frac{5^2}{2^5} = \frac{25}{32}$$

(b)

$$0.781$$

Question 2

$$\sin 5 = \frac{h}{3.17}$$
$$h = 3.17 \sin 5$$
$$= 0.276$$

Question 3

(a)

$$\frac{8}{500} = 0.016kg$$

(b)

$$1.6 \times 10^{-2} kg$$

Question 4

$$1$$

in passing, it is worh mentioning that $\cos^2 x + \sin^2 x = 1$ *is a well known identity*

Question 5

(a)

$$3$$

(b) 3 lines drawn through center of each shape, i.e. each line goes through two shapes - start off with a vertical line through the center of the entire figure

Question 6

(a)

$$PB = BQ = 4$$

using Pythagoras on $\triangle BPQ$

$$PQ^2 = 4^2 + 4^2$$
$$PQ^2 = 32$$

so

$$PQ = 5.657$$

(b) Area of PQRS

$$= PQ^2 = 32, \quad \text{from above}$$

Question 7

(a)

$$21.5 \leq d < 22.5$$

(b)

$$21.5 \times 5 \times 1.6 = \$172.00$$

Question 8

$$\frac{1}{2}x + 2y = 16 \qquad (1)$$
$$2x + \frac{1}{2}y = 19 \qquad (2)$$

(1) $\times 4$

$$2x + 8y = 64 \qquad (3)$$

(3 - (2)

$$7.5y = 45$$
$$y = \frac{45}{7.5} = 6$$

Into (1)

$$\frac{1}{2}x + 2(6) = 16$$
$$\frac{1}{2}x = 4$$
$$x = 8$$

Question 9

(a)

$$w \ \alpha \ \frac{1}{f}$$

as an equation

$$w = \frac{k}{f}$$

where k is a constant

When f=200, w=1500

$$1500 = \frac{k}{200}$$
$$k = 1500 \times 200$$
$$= 300000$$

so equation becomes

$$w = \frac{300000}{f}$$

(b) *transposing equation*

$$f = \frac{300000}{w}$$

when $w = 600$

$$f = \frac{300000}{600} = 500$$

Question 10

(a) From 1 to 19, the prime numbers are

$$2, 3, 5, 7, 11, 13, 17, 19$$

so required probability

$$= \frac{8}{19}$$

(b)

$$\frac{7}{18}$$

Question 11

$n(A) = 12, n(A \cup B) = 19 \Rightarrow$ B itself, excluding intersection with A, alone contains 7

$n(\xi) = 21, n(A \cup B) = 19 \Rightarrow$ elements totally outside circles = 2

$n(A \cap B') = 8$ with above $\Rightarrow$ A itself, excluding intersectyion, alone contains 8

$n(A) = 12$ so from previous line $n(A \cap B) = 4$

Question 12

(a)

$$2\mathbf{M} = \begin{bmatrix} 2x & 4x \\ 4x & 2x \end{bmatrix}$$

(b)

$$\mathbf{M}^2 = \begin{bmatrix} x & 2x \\ 2x & x \end{bmatrix} \begin{bmatrix} x & 2x \\ 2x & x \end{bmatrix}$$

$$= \begin{bmatrix} x^2 + 4x^2 & 2x^2 + 2x^2 \\ 2x^2 & 4x^2 + x^2 \end{bmatrix}$$

$$= \begin{bmatrix} 5x^2 & 4x^2 \\ 4x^2 & 5x^2 \end{bmatrix}$$

Question 13

(a)

2	3	4
8	11	14

(b) The first differences are constant and have the value 3, implying the formula is of the form

$$3n + k$$

comparing 3n (i.e. 3,6,9,...) with the original sequence

$$\Rightarrow k = 2$$

so general formula

$$= 3n + 2$$

(c)

$$3(60) + 2 = 182$$

(d) if

$$3n + 2 = 89$$

$$3n = 87$$

$$n = 29$$

Question 14

(a) as a fraction

$$\frac{62000}{310000} = \frac{62}{310}$$

as a percentage

$$\frac{62}{310} \times 100 = 20\%$$

(b)

$$500\%$$

either using a similar calculation to (a)

$$\frac{310}{62} \times 100 = 500$$

or else realise that it is the 'reciprocal' of the answer from (a)

Question 15

(a) Gradient

$$= \frac{5-2}{8-6} = \frac{3}{2}$$

(b) general form of equation is

$$y - y_0 = \frac{3}{2}(x - x_0)$$

Insert the coordinates (6,2)

$$y - 2 = \frac{3}{2}(x - 6)$$

$$y = \frac{3}{2}x - 9 + 2$$

$$y = \frac{3}{2}x - 7$$

Question 16

$$\frac{x+2}{x} - \frac{x}{x+2}$$

$$= \frac{(x+2)^2 - x^2}{x(x+2)}$$

$$= \frac{x^2 + 4x + 4 - x^2}{x(x+2)}$$

$$= \frac{4x + 4}{x(x+2)}$$

Question 17

(a)

$$h = 3\sin(0) = 0$$

(b)

$$h = 3\sin(30 \times 7)$$

$$= 3\sin(210)$$

$$= -1.5$$

(c) h indicates 1.5 units <u>below</u> that of midday

Question 18

(a) In the first 'column', the missing probability is 0.4

In the second 'column', the missing probability in both cases is 0.3

(b) two branches of tree correspond to this scenario - multiply along each branch

$$0.6 \times 0.3 = 0.18$$

$$0.4 \times 0.7 = 0.28$$

P(passes only one of the two tests)

$$= 0.18 + 0.28 = 0.46$$

Question 19

(a)

$$0.2x + 3.6 = 1.2$$

$$0.2x = -2.4$$

$$x = -12$$

(b)

$$\frac{2-3x}{5} < x + 2$$

$$2 - 3x < 5x + 10$$

$$-8 < 8x$$

$$8x > -8$$

$$x > -1$$

Question 20

(a) required bearing

$$= 115 + (180 - 63) = 232^\circ$$

(b) Using sine rule, and letting $\angle GAW = x$

$$\frac{\sin x}{400} = \frac{\sin 63}{410}$$

$$\sin x = \frac{400 \sin 63}{410} = 0.869$$

$$x = 60.374$$

so bearing of W from A

$$= 115 + 60.374$$

$$= 175.374$$

so bearing of A from W

$$= 175.374 + 180 = 355.374$$

$$= 355^\circ \text{ to 3 sig fig}$$

Question 21

(a) (i)

$$20^\circ$$

angle subtended at the circumference is half that subtended at the center

(ii)

$$\angle OAB = \frac{1}{2}(180 - 40)$$

$$= \frac{1}{2}(140)$$

$$= 70^\circ$$

making use of the fact that $\triangle OAB$ is isosceles

(b) (i)

$$\frac{40}{360} \times 2\pi r$$

$$= \frac{1}{9} \times 2\pi(5)$$

$$= \frac{10\pi}{9}$$

$$= 3.491cm$$

(ii)

$$\frac{40}{360} \times \pi r^2$$

$$= \frac{1}{9} \times \pi(5)^2$$

$$= \frac{25\pi}{9}$$

$$= 8.727cm^2$$

Paper 3 - 0580/03 - 0581/03 , May/June 2005

Question 1

(a)

$$2.8kg$$

(b)

$$150 \times 0.71 = \pounds\mathit{106.50}$$

(c) (i)

$$1040$$

(ii)

$$1h\ 30m$$

(d)

$$1140 + 2h\ \ 15m$$

$$= 1355$$

(e)

$$420 \times \frac{85}{100} = 357$$

or could use 0.85 instead of the fraction

Question 2

(a)

$$x = -1 \Rightarrow y = 1 + 2(-1) - (-1)^2 = 1 - 2 - 1 = -2$$

$$x = 0 \Rightarrow y = 1$$

$$x = 1 \Rightarrow y = 1 + 2(1) - (1)^2 = 1 + 2 - 1 = 2$$

$$x = 4 \Rightarrow y = 1 + 2(4) - (4)^2 = 1 + 8 - 16 = -7$$

(b) Draw graph - a parabola with maximum at $y = 2$

(c)

$$x = -0.4, 2.4$$

or similar

where the line crosses the x-axis

(d) (i) line of symmetry is vertical line with x-coordinate midway betwen the points where the line crosses the x-axis, i.e. x= 1

(ii)

$$x = 1$$

Question 3

(a) (i) min : -3
max : 9

(ii)

$$7 - (-2) = 7 + 2 = 9$$

(b) complete graph

(c) (i) 3

(ii) Sunday

(d)

$$F = \frac{9C}{5} + 32$$

when $C = 6$

$$F = \frac{9 \times 6}{5} + 32$$

$$= \frac{54}{5} + 32$$

$$10.8 + 32$$

$$= 42.8^\circ$$

Question 4

(a) (i)

$$\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

(ii) move all vertices 3 units to the right and 1 unit downwards (i.e. coordinate units)

(b) (i)

$$\begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

(ii) move all vertices 2 units to the left and 2 units upwards

(c) enlargement by a scale factor of 2 about origin

(d) (i)

$$1$$

vertical line coincident with y-axis

(ii)

$$1$$

(iii) new vertices at

$$(-2, -1),\ \ (2, 1),\ \ (0, 3)$$

(iv) reflection about $y = 0$ (the x-axis)

Question 5

(a) (i)

Number	*Frequency*
1	8
2	7
3	10
4	9
5	8
6	18

(ii)

$$6$$

(iii) *because there is an even number of data items, median is* $\frac{1}{2}$ *(30th item + 31st item)*

$$4$$

both 30th and 31st items have the same value

(iv) total of scores

$$= (1\times8)+(2\times7)+(3\times10)+(4\times9)+(5\times8)+(6\times18)$$

$$= 8 + 14 + 30 + 36 + 40 + 108$$

$$= 236$$

so mean

$$= \frac{236}{60} = 3.933 = 3.9 \text{ to 1 d.p.}$$

(b) (i)

$$10 + 7 + 10 + 7 + 14 + 12$$

$$= 60$$

(ii) total of scores

$$(1\times10)+(2\times7)+(3\times10)+(4\times7)+(5\times14)+(6\times12)$$

$$= 10 + 14 + 30 + 28 + 70 + 72$$

$$= 224$$

mean

$$= \frac{224}{60} = 3.733$$

Question 6

(a) (i)

$$2x + x + 2x + x = 36$$

$$6x = 36$$

$$x = \frac{36}{6} = 6$$

(ii) Area

$$= 12 \times 6 = 72\ cm^3$$

(b) (i)

$$3y = y + 3$$

$$2y = 3$$

$$y = \frac{3}{2}$$

(ii)

$$4z + 2 = 10z - 1$$

(iii)

$$6z = 3$$

$$z = \frac{3}{6} = \frac{1}{2}$$

(c) (i)

$$4a + b = 17 \quad (4)$$

$$a - b = 3 \quad (5)$$

(ii) adding two equations above

$$5a = 20$$

$$a = 4$$

into (5)

$$4 - b = 3$$

$$b = 1$$

Question 7

(a)

$$050$$

(b) (i) draw a line from B at an angle of 10° west of N

(ii) mark intersection of this line with AC

(c) (i) 6.6 cm

(ii)

$$6.6cm : 14km$$

$$6.6 : 1400000$$

$$1 : 212121.212$$

(d) (i) in centimeters

$$\frac{10}{212121.212} = 0.00004714$$

$$= 4.714cm$$

using compasses, draw arc of 4.7 cm from A

(ii) Draw SR and note where it intersects with the arc from (i). Measuring the distance from this point to R

$$2.3cm$$

or similar

and

$$2.3 \times 212121.212$$

$$= 487878.788cm$$

$$= 4.878...km$$

$$= 4.88 \text{ to 3 sig figs}$$

(e) (i) BR measures

$$9cm$$

or similar

and

$$9 \times 212121.212$$
$$= 1909090.908cm$$
$$= 19.1km \text{ to 3 sig figs}$$

(ii)

$$19.1km \ \ in \ \ 40 \ \ mins$$
$$= \frac{19.1}{40} \times 60 = 28.65km/h$$
$$= 28.7km/h \quad \text{to 3 sig figs}$$

(iii)

$$\frac{28.65}{1.85} = 15.486 \ \ knots$$

Question 8

(a) 2 sides of $4 \times 6 \Rightarrow 24 \times 2 = 48$
2 sides of $4 \times 8 \Rightarrow 32 \times 2 = 64$
2 sides of $6 \times 8 \Rightarrow 48 \times 2 = 96$

therefore total area $= 208cm^2$

(b) Volume

$$= 4 \times 6 \times 8 = 192cm^3$$

(c) (i) draw straight line from A to C

(ii) Using Pythagoras

$$AC^2 = 8^2 + 10^2$$
$$= 64 + 100$$
$$= 164$$

so

$$AC = 12.806cm$$

(iii)

$$\tan CAB = \frac{10}{8} = 1.2$$
$$\angle CAB = 50.194 = 50.2^\circ \text{ to 1 d.p.}$$

Paper 4 - 0580/04 - 0581/04 , May/June 2005

Question 1

(a)

$$\frac{1.33}{7} \times 5 = 0.950\ tonnes = 950kg$$

(b)

$$765 \times \frac{9}{17} = \$405$$

(c)

$$\frac{405}{950} = \$0.426/kg$$

(d) (i)

$$0.35 \times 0.6 = \$0.21/kg$$

(ii)

$$\frac{0.35}{125} \times 100 = \$0.28/kg$$

Question 2

(a) draw horizontal line of length 12

(b) Using compasses, with the point on each end of the line in turn, construct arcs above and below line, arcs which intersect. Construct single line through both intersections, this being the perpendicular bisector

(c) Mark off 14 cm on your perpendicular bisector. Then bisect this 'perpendicular bisector' itself using the methods just described in the first paragraph.

Mark off 4.5 cm either side of this new line, to produce a side of 9 cm long (as in the diagram).

Connect your two lines to form the sides of then trapezium.

(d)

$$78°$$

or similar

(e) Consider a triangle 'chopped off' the side of your trapezium. By symmetry, this will have sides of 1.5cm, 7cm plus the side of the quadrilateral of unknown length If we let

$$\tan ABC = \alpha$$

then

$$\tan\alpha = \frac{7}{1.5}$$
$$= 4.667$$

so

$$\alpha = 77.905$$
$$= 77.9° \text{ to 1 d.p.}$$

(f) (i) use compasses to form arc 5cm from D

(ii) bisect $\angle ADC$. Using compasses, with point on D construct arcs on DC and DA. From each of these arcs in turn, construct arcs which intersect. Connect this intersection with D

(iii) shade the lower part of area inside the arc i.e. area separated off by the bisector

Question 3

(a) (i) translation of 6 units in negative x-direction, and 1 unit in positive y-direction.

(ii) reflection about $y = -x$

(iii) enlargement by factor of 3 about $(0, 6)$

(iv) shear with y coordinates constant

(b) (i)

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$$

Question 4

(a)

$$f = 2^x$$

$$x = -2 \rightarrow f = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$
$$x = 0 \rightarrow f = 2^0 = 1$$
$$x = 3 \rightarrow f = 2^3 = 8$$

(b) graph line using figures given

(c)

$$2.8$$

or similar

x value corresponding to 7 on vertical axis

(d)

$$f(x) = 2^{-\infty} = \frac{1}{2^{\infty}} = 0$$

(e)

$$2$$

or similar

(f) draw straight line $y = 2x + 1$, a line crossing the y-axis at $y = 1$, and with a gradient of 2

(g) *the intersection of the two lines gives x= 0, 2.8 (or similar)*

Thus 2.8 is the required non-integer solution

Question 5

(a) (i)

$$\vec{DC} = \mathbf{c} - \mathbf{d}$$

(ii)

$$\vec{OE} = \frac{1}{2}\vec{CO} + \vec{CD}$$

$$\vec{OE} = \frac{1}{2}\mathbf{c} + (-(\mathbf{c} - \mathbf{d}))$$

$$= \frac{1}{2}\mathbf{c} - (\mathbf{c} - \mathbf{d})$$

$$= -\frac{1}{2}\mathbf{c} + \mathbf{d}$$

(iii)

$$\vec{OB} = \frac{1}{2}\vec{OC} + \vec{DC}$$

$$= \frac{1}{2}\mathbf{c} + (\mathbf{c} - \mathbf{d})$$

$$= \frac{3}{2}\mathbf{c} - \mathbf{d}$$

(b) (i) angles in hexagon total

$$(n-2) \times 180$$

$$= 4 \times 180$$

$$= 720$$

each angle

$$= \frac{720}{6} = 120^\circ$$

(and this will therefore be the size of $\angle ABC$)

(ii) Area

$$= \frac{1}{2} \times 8 \times 8 \times \sin 120$$

$$\frac{1}{2} \times 64 \times \frac{\sqrt{3}}{2}$$

$$= 27.713cm^2$$

$$= 27.7cm^2 \text{ to 3 sig fig}$$

(iii)

$$AC^2 = CB^2 + AB^2 - (CB)(AB)\cos 120$$

$$= 8^2 + 8^2 - (8)(8)\left(-\frac{1}{2}\right)$$

$$= 64 + 64 + 32$$

$$= 160$$

so

$$AC = 12.649cm$$

$$= 12.6cm \text{ to 3 sig figs}$$

(iv) Area of ACDO

$$= (8)(8 \times \cos 30 \times 2)$$

$$= 110.851$$

so area of hexagon

$$110.851 + 2(27.713) = 166.277cm$$

$$= 166cm^2 \text{ to 3 sig figs}$$

by splitting the hexagon into a rectangle and two triangles. Each 'triangle' will be isosceles allowing you to compute the relevant angle of 30 degrees used in the above formula. If you are unsure how I have calculated the longer side of ACDO, then resort to using basic trigonometry on the 'triangles' to compute the relevant length

Question 6

(a) Volume of cylinder

$$\pi r^2 h = \pi(0.35)^2 \times 16.5$$

$$= 6.35$$

Volume of cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(0.35)^2(1.5)$$

$$= 0.192$$

Volume of pencil

$$= 6.35 + 0.192 = 6.542cm^2$$

$$= 6.54cm \text{ to 3 sig figs}$$

(b) (i)

$$w = 6 \times 0.7 = 4.2cm$$

$$x = 2 \times 0.7 = 1.4cm$$

(ii) Volume of box

$$= 4.2 \times 1.4 \times 18 = 105.84cm^2$$

(iii) as fraction

$$\frac{6.542 \times 12}{105.84}$$

as percentage

$$\frac{6.542 \times 12}{105.84} \times 100$$

$$= 74.172$$

$$= 74.2\% \text{ to 3 sig figs}$$

(c) (i) Using Pythagoras

$$l^2 = 1.5^2 + 0.35^2$$

$$l = 1.54cm$$

(ii) area of cone

$$A = \pi rl$$
$$= \pi(0.35)1.54$$
$$= 1.693$$

area of cylinder

$$= \pi r^2 + 2\pi rh$$
$$= \pi(0.35)^2 + 2\pi(0.35)(16.5)$$
$$= 36.670$$

note : the side of the cyclinder can be considered flat and calculated in the appropriate manner - think of peeling the label off a food can and the way it can be turned out flat

total area

$$= 1.693 + 36.670 = 38.363$$
$$= 38.4cm^2 \text{ to 3 sig figs}$$

Question 7

(a) (i)

$$46.5km/h$$

corresponding to 75

(ii) Lower quartile $= 42km/h$
Upper quartile $= 51.5km/h$

so inter-quartile range

$$= 51.5 - 42$$
$$= 9.5$$

(iii)

$$150 - 102 = 48$$

(b) (i)

$$32$$

in order to total 150

(ii) total of speeds

$$(32.5 \times 10) + (37.5 \times 17) + (42.5 \times 33) + (47.5 \times 42)$$
$$+(52.5 \times 32) + (57.5 \times 16)$$
$$= 6960$$

mean

$$= \frac{6960}{150} = 46.4km/h$$

(c) Divide frequency by spread of speed to get the required height

30-40 : 2.7
40-55 : 7.133 (drawn as 7.1)
55-60 : 3.2

this figures on the right will be the height of each column

Question 8

(a)

$$x^2 - 4x + 3 = 0$$
$$(x-1)(x-3) = 0$$

so

$$x - 1 = 0 \Rightarrow x = 1$$
$$x - 3 = 0 \Rightarrow x = 3$$

(b)

$$g(x) = 2x - 1$$

Let

$$y = 2x - 1$$
$$2x = y + 1$$
$$x = \frac{y+1}{2}$$

altering letters

$$g^{-1}(x) = \frac{x+1}{2}$$

the function on the penultimate line is a function of y - the variable names are then swopped around to produce a function of x, which is asked for. It is important to realize that

$$h(y) = \frac{y+1}{2}$$

and

$$h(x) = \frac{x+1}{2}$$

represent the same function. What the variable is called is unimportant, per se

(c)

$$x^2 - 4x + 3 = 2x - 1$$
$$x^2 - 6x + 4 = 0$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2}$$
$$= \frac{6 \pm \sqrt{36 - 16}}{2}$$
$$= \frac{6 \pm \sqrt{20}}{2}$$
$$= 3 \pm \frac{\sqrt{20}}{2}$$
$$= 0.764, 5.236$$
$$= 0.76, 5.24 \text{ to 2 dp}$$

(d)

$$gf = 2(x^2 - 4x + 3) - 1$$
$$= 2x^2 - 8x + 6 - 1$$
$$= 2x^2 - 8x + 5$$

$$gf(-2) = 2(-2)^2 - 8(-2) + 5$$
$$= 8 + 16 + 5$$
$$= 29$$

(e)

$$fg = (2x-1)^2 - 4(2x-1) + 3$$
$$= 4x^2 - 2x - 2x + 1 - 8x + 4 + 3$$
$$= 4x^2 - 12x + 8$$

Question 9

(a)

$$x + y \leq 12$$

(b)

$$y \geq 4$$

(c) draw axes

(d) inequalities become

$$y \geq -\frac{5}{3}x + 15$$
$$y \leq -x + 12$$
$$y \geq 4$$

replace inequalities with equal signs and draw relevant straight lines

for each line, examine whether the corresponding inequality refers to above or below the line, and therefore shade off the *unwanted* regions in each case.

Finally, the region to be examined is that with vertices

$$(4.2, 7.5), (6.6, 4), (8, 4)$$

or similar

(e)

$$Cost = 20x + 10y$$

examing each vertex

At $(4.2, 7.5)$

$$Cost = 20(5) + 10(7) = 170$$

At $(6.6, 4)$

$$Cost = 20(6) + 10(5) = 170$$

At $(8, 4)$

$$Cost = 20(8) + 10(4) = 200$$

So two possibilities are

5 SUPER and 7 MINI
or
6 SUPER and 5 MINI

since two of the vertices do not fall on discrete values, examine relevant points on the boundary immediately adjacent to the relevant vertices to find the optimal solution

(f) (i) Two points on the boundary correspond to 11 taxis (7,4) and (6,5)

$$Charges = 30x + 16y$$

so

$$30(7) + 16(4) = 274$$
$$30(6) + 16(5) = 260$$

(ii) combing the two functions

$$Profit = 10x + 6y$$

so

$$10(7) + 6(4) = 94$$
$$10(6) + 6(5) = 90$$

so
7 SUPER and 4 MINI gives a profit of \$94

Paper 1 - 0580/01 - 0581/01 , October/November 2005

Question 1

$$1.01 \times 10^4$$

Question 2

$$x(3y-2)$$

and check by working backwards to see whether you regain the original expression

Question 3

$$6950m$$

The true height is given by

$$6950 \leq \; real \;\; height < 6970$$

Question 4

$$\sqrt{5}$$

a rational number is a number that can be represented by a fraction. Three of the five given nunbers can obviously be represented in this way. $\sqrt{81}=9$, thus $\sqrt{5}$ is the only one left

Question 5

$$5x-7=8$$
$$5x=15$$
$$x=3$$

Question 6

Could do this using mental arithmetic

To do it formally

$$1\frac{1}{2} \div \frac{1}{8}$$
$$\frac{3}{2} \div \frac{1}{8}$$
$$\frac{3}{2} \times \frac{8}{1}$$
$$=12$$

Question 7

(a)

$$1-(-9)=10^\circ$$

(b)

$$-9+21=12^\circ$$

Question 8

$$P=2a+2b$$
$$2a=P-2b$$
$$a=\frac{P-2b}{2}$$

Question 9

Converting all to decimals they become (in the same order)

$$0.072, \;\; 0.72, \;\; 0.702, \;\; 0.7, \;\; 0.07, \;\; 0.072$$

so

(a)

$$\frac{7}{100}$$

(b)

$$72\%$$

(c)

$$0.072 \;\; and \;\; 7.2\%$$

Question 10

(a)

$$61 \;\; or \;\; 67$$

only one needs to be given

(b)

$$63$$

(c)

$$64$$

Question 11

(a)

$$\mathbf{p}+\mathbf{q}=\begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

(b) $\vec{OP}$ will be an arrow with its tail at the origin and its tip on $(2,-3)$

Question 12

$$\sin 21^\circ = \frac{height}{1.2}$$
$$height \; = 1.2\sin 21^\circ$$
$$= 0.430km \text{ to 3 sig figs}$$
$$= 430m$$

Question 13

The most elegant way to do it would be to first calculate the decrease, i.e. 200,000

The decrease as a fraction

$$= \frac{200,000}{2,700,000}$$

$$= \frac{2}{27}$$

as a percentage

$$= \frac{2}{27} \times 100 = 7.407...$$

$$= 7.41 \text{ to 3 sig figs}$$

Question 14

(a)

$$x > y$$

$6 > -7$

(b)

$$x^2 < y^2$$

$36 < 49$

(c)

$$y - x < x - y$$

$-13 < +13$

Question 15

(a)

$$0.5$$

do not write 0.50 *as this would lose you marks*

(b) (i)

$$9.9 - 5.8 \times 0.5$$

$$= 9.9 - 2.9$$

$$= 7$$

remember that multiplications need to be done before additions

(ii)

$$7.0908$$

Question 16

(a)

$$3r - 3s$$

(b)

$$q$$

(c)

$$p^4$$

Question 17

(a) Cleopatra has a fractional share of

$$\frac{5}{20} = \frac{1}{4}$$

so required total

$$= 15 \times 4 = \$60$$

(b) Dalila has a share of

$$\frac{7}{20} \; of \; \$60$$

$$= \frac{7}{20} \times 60$$

$$= \$21$$

so

$$21 - 12 = \$9$$

Question 18

(a)

$$400 - (2 \times 120)$$

$$= 400 - 240$$

$$= 160m$$

(b) Two ends form a circle of diameter d

So using

$$C = \pi d$$

$$d = \frac{C}{\pi}$$

$$= \frac{160}{\pi}$$

$$= \; 50.9 \text{ to 3 sig figs}$$

Question 19

(a)

$$45 \times 0.65 = \$29.25$$

(b)

$$\frac{45}{2.5} = 18$$

(c)

$$\frac{29.25}{2.20} = 13.29....$$

so in order to make a profit he needs to sell 14 bags

Question 20

(a)

$$35\% \ \ of \ \ \$900$$

$$= \frac{35}{100} \times 900$$

$$= 35 \times 9$$

$$= \$315$$

(b)

$$12 \times 60 = \$720$$

so total payments

$$= 720 + 315$$

$$= \$1035$$

which exceeds \$900 by \$135

Question 21

(a)

$$\$62$$

(b)

$$2.5 \ hours$$

(c) (i) draw line thru

$$(0,0),\ (1,16),\ (2,32),\ (3,48),\ (4,64),\ (5,80)$$

(ii)

$$5 \ hours$$

Paper 2 - 0580/02 - 0581/02 , October/November 2005

Question 1

$$T = \frac{1}{2}(20)(21)$$
$$= 10(21)$$
$$= 210$$

Question 2

$$2(\sin 15)(\cos 15)$$
$$= 2 \times \frac{1}{4}$$
$$= \frac{1}{2}$$

Question 3

$$(4 \times 3) + (6 \times 2) + (2 \times (-12))$$
$$= 12 + 12 - 24$$
$$= 0$$

Question 4

(a)

$$-1.8$$

sequence drops by 2 every term

(b)

$$21$$

sequence increases by 2,3,4,5, etc

Question 5

$$800 \times \frac{3}{100} \times \frac{5}{12}$$
$$= \$10$$

Question 6

Might need to simplify $(0.8)^{-1}$, i.e.

$$(0.8)^{-1} = \left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$$

(a)

$$(0.8)^2$$

bearing in mind that squaring a number less than 1 will produce a number of lower magnitude, whereas squaring root a number less than one will produce a number of greater magnitude

(b)

$$(0.8)^{-1}$$

as already shown this is greater than one - all the other figures will be less than one

Question 7

(a)

$$f(0.5) = 10^{0.5} = 3.16 \text{ to 3 sig figs}$$

this is the square root of 10

(b) Let

$$y = 10^x$$
$$\log y = \log(10^x)$$
$$\log y = x$$

to make the 'inverse' nature of this function more obvious could state it as

$$x = \log y$$

so

$$f^{-1}(y) = \log y$$

and

$$f^{-1}(1) = \log 1 = 0$$

Question 8

$$\vec{OB} = \mathbf{c} + \mathbf{a}$$

so

$$\vec{OM} = \frac{1}{2}(\mathbf{c} + \mathbf{a})$$

now

$$\vec{MA} = \vec{OA} - \vec{OM}$$
$$= \mathbf{a} - \frac{1}{2}(\mathbf{c} + \mathbf{a})$$
$$= \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{c}$$

or

$$\frac{1}{2}(\mathbf{a} - \mathbf{c})$$

Question 9

(a)

$$2380$$

(b)

$$2381.60$$

(c)

$$2400$$

Question 10

Mass of Saturn

$$= 5.97 \times 10^{24} \times 95$$
$$= 567.15 \times 10^{24}$$
$$= 5.6715 \times 10^{26}$$
$$= 5.7 \times 10^{26} \text{ to 2 sig figs}$$

note on third line, the mantissa (the left-hand term) has been reduced by a factor of 100, so the power term (on the right hand side) is increased by a factor of 100

Question 11

The four rectangular sections have a total length of

$$4 \times 4 = 16m$$

The four corner sections form a single circle of radius 1.2m. Its circumference would be given by

$$C = 2\pi r$$

$$= 2\pi(1.2)$$

$$= 7.539......$$

So total length of perimeter is

$$16 + 7.539... = 23.539...m$$

So the greatest number of people who can sit around the table

$$= 23$$

Question 12

$$c = \frac{d^3}{2} + 5$$

$$2c = d^3 + 10$$

$$d^3 = 2c - 10$$

$$d = \sqrt[3]{2c - 10}$$

Question 13

$$F \ \alpha \ \frac{1}{d^2}$$

so

$$F = \frac{k}{d^2}$$

and therefore

$$30 = \frac{k}{4^2} \Rightarrow k = 30 \times 16 = 480$$

and eqn becomes

$$F = \frac{480}{d^2}$$

When $d = 8$

$$F = \frac{480}{8^2} = 7.5$$

Question 14

(a)

$$7a(c + 2)$$

(b)

$$6ax(2x^2 + 3a^2)$$

Question 15

(a)

$$180 - 26 = 54°$$

since CF and AD are parallel, use supplementary angles

(b) Using

$$\angle ABC = 90$$

(because it is the angle subtended by a diameter)

$$\angle EBC = 90 - 48 = 42°$$

(c)

$$\angle AEB = 180 - 126 = 54°$$

using complementary angles

so

$$\angle BAE = 180 - (48 + 54) = 78°$$

Question 16

$$4 - 5x < 2(x + 4)$$

$$4 - 5x < 2x + 8$$

$$-4 < 7x$$

$$-\frac{4}{7} < x$$

Question 17

(a) $\angle AEF$ is an exterior angle and exterior angle of a pentagon is given by

$$\frac{360}{5} = 72°$$

(b) $\triangle DAE$ is isosceles. So

$$\angle DAE = \frac{1}{2}(180 - 108) = \frac{1}{2}(72) = 36°$$

Question 18

(a)

$$\left(\frac{x^9}{3}\right)^2 = \frac{x^{18}}{9}$$

(b)

$$\left(\frac{x^{-2}}{4}\right)^{-\frac{1}{2}}$$

$$= \left(\frac{1}{4x^2}\right)^{-\frac{1}{2}}$$

$$(4x^2)^{\frac{1}{2}}$$

$$= 2x$$

Question 19

(a)

$$\frac{(5-0)}{0-10} = \frac{5}{-10} = -\frac{1}{2}$$

(b) y-intercept = 5

so equation is given by

$$y = -\frac{1}{2}x + 5$$

Question 20

(a) Area of triangle (from t=0 to t=3)

$$= \frac{1}{2} \times 12.4 \times 3 = 18.6$$

Area of rectangle (from t=3 to t=8)

$$= 12.4 \times 5 = 62$$

So total distance required

$$= 18.6 + 62 = 80.6m$$

(b) Area corresponding to times between t=8 and t=10

$$= 100 - 80.6 = 19.4m$$

Let final speed = x

Then equating area under line (between t=8 to t=10) to 19.4
(and splitting the said area into a rectangle and a triangle)

$$2x + \frac{1}{2}(2)(12.4 - x) = 19.4$$
$$2x + 12.4 - x = 19.4$$
$$x = 7m/s$$

Question 21

(a)

$$AE^2 + 4^2 = 8^2$$
$$AE^2 = 64 - 16 = 48$$

so

$$AE = 6.9282....$$
$$= 6.93 \text{ to 3 sig figs}$$

(b) Area of ABCD

$$= 2AE \times 8$$
$$= 110.8512....$$

Area of circle radius 4

$$= \pi(4)^2$$
$$= 50.2654....$$

Therefore area of shaded region

$$= 60.5858...$$
$$= 60.6 \text{ to 3 sig figs}$$

Question 22

(a) (i) With compasses on C, construct two arcs intersecting BC and AC. With the compasses on each of these intersections in turn construct intersecting arcs. Draw a line between C and this latter intersection

(ii) With compasses set to more than half the distance of AC, place the compasses in turn on A and C and construct arcs above and below the line that intersect. Draw a line between these two intersections.

(b) With compasses measured to 7cm, place point on C and draw arc confined to interior of triangle

(c) Use the three lines just constructed. Those points nearer to A than C are to the 'left' of the perpendicluar bisector of AC. Those nearer to BC than AC are those points below the bisector of angle ACB. And those less than 7cm from C are those to the 'right' of the arc from (b). Identify those points that satisfy all three conditions.

Question 23

(a)

$$\frac{5x}{2} - 9 = 0$$
$$\frac{5x}{2} = 9$$
$$5x = 18$$
$$x = \frac{18}{5}$$
$$= 3.6$$

(b)

$$x^2 + 12 + 3 = 0$$
$$(x+6)^2 - 6^2 + 3 = 0$$
$$(x+6)^2 = 33$$
$$x + 6 = \pm\sqrt{33}$$
$$x = \pm\sqrt{33} - 6$$

so

$$x - 0.255..$$
$$= -0.3 \text{ to 1 dp}$$

or

$$x = -11.7 \text{ to 1 dp}$$

or alternatively use the formula

Paper 3 - 0580/03 - 0581/03 , October/November 2005

Question 1

(a) Treat the line m as a mirror, so that every vertex and vertical line of the E is placed an equal distance on the other side of this line

(b) (i) rotation of 90° counter-clockwise about the point on the very top line of the graph common to both figures

(ii) Enlargement with scale factor of 3 about a bottom on the very bottom line of the graph, a half square in from the right

Note the importance in each of the above of giving **all** *the necessary information, e.g. relevant points/lines and scale factors*

(iii) translation described by

$$\begin{bmatrix} -7 \\ -5 \end{bmatrix}$$

Question 2

(a) (i)

$$\tan ABC = \frac{6}{4}$$

$$ABC = \tan^{-1} 1.5 = 56.3 \text{ to 3 sig figs}$$

(ii)

$$\angle CBD = 180 - 56.3 = 123.7°$$

(b)

$$BC^2 = 6^2 + 4^2$$

$$= 36 + 16$$

$$= 52$$

so

$$BC = 7.21 \text{ to 3 sig figs}$$

(c) Perimeter

$$= 6 + 4 + 7.21 = 17.2m \text{ to 3 sig figs}$$

Area

$$= \frac{1}{2}(6)(4) = 12m^2$$

Question 3

(a) (i) Added figures are

x	y
-2	5
0	-3
5	12

(ii) Draw graph

(iii) Find the intersection between the graph you have just drawn and $y = -1$

Giving

$$x = -0.8, \quad x = 2.6$$

or close to this figure

(b) (i) Added figures are

x	y
0.25	8
1	2

(ii) Draw graph

(iii)

$$3.1$$

or close to this figure

Question 4

(a)

8.3571..........

$$= 8.36 \text{ to 3 sig figs}$$

(b) When arranged in order, the 7th term = 6 and the 8th term = 10

So the median

$$= \frac{1}{2}(10 + 6) = 8$$

(c)

6

(d)

No of calls	0 − 4	5 − 9	10 − 14	15 − 19
Frequency	3	4	4	3

(e) (i)

$$\frac{7}{14}$$

(ii)

$$\frac{3}{14}$$

(f) Prob of 10-14 calls

$$= \frac{4}{14} = \frac{2}{7}$$

Estimate of no. of calls in next six weeks

$$= \frac{2}{7} \times 42 = 12$$

Question 5

(a) Draw a line 6cm long from A at angle of 100° between this line and the North line. Draw another line from A at a further 110° from the first line (as in the diagram). Connect the ends of these two lines.

(b) (i)

$$39$$

or close to this figure

(ii) somewhere between

$$247^\circ - 250^\circ$$

remember to measure the angle counter-clockwise from North. Use a set square and straight edge to draw a North line at C, i.e. to produce a line parallel to the given North line at A.

(c)

$$9km$$

d) (i) draw an arc of 4cm from A and an arc 5cm from C, such that these two arcs intersect in two places, which are marked as P and Q.

(ii) Use compasses set to 6cm to draw an arc from B, such that only one of P and Q will be closer to B than this particular arc.

$$4.0km \ \ to 4.4km$$

Question 6

(a) (i) Breaking the cross-section into a rectangle and a trapezium

Area of rectangle

$$= 2 \times 3.6 = 7.2m^2$$

Area of trapezium

$$= (2.4)\left(\frac{2+1}{2}\right)$$
$$= (2.4)(1.5)$$
$$= 3.6$$

Total

$$= 7.2 + 3.6 = 10.8m^2$$

(ii) Volume

$$= 10.8 \times 3 = 32.4m^3$$
$$= 32400 \ liters$$

(iii) Area of surface

$$= 3 \times 6 = 18m^2$$

In one hour, evaporation $= 18 \ liters$

In two hours, evaporation

$$= 18 \times 2 = 36 \ \ liters$$

(b) (i) Time

$$= \frac{61500}{1000} = 61.5 \ \ hours$$
$$= 61 \ \ hours \ \ \ 30 \ \ mins$$

(ii)

$$\frac{61500}{4.55} = 13516.48 \ \ galls \ \text{ to 2 dp}$$

given the injunction to state answers to three significant figures, 13500 would be acceptable

(iii)

$$\frac{13516}{10000} \times 2.5 = 3.379 = 3.38 \ \text{ to 3 sig figs}$$

(iv) 4 bottles

Question 7

(a) (i)

$$2x - y = 3$$

(ii)

$$2$$

from form y=2x-3

(iii)

$$\begin{bmatrix} x & -1 & 0 & 1 & 2 & 3 \\ y & 3 & 2 & 1 & 0 & -1 \end{bmatrix}$$

(iv) draw graph

(v) Need coordinates of intersection between two lines, i.e.

$$(1.6, 0.3)$$

or close figure

(b)

$$2x - y = 3$$
$$x + y = 2$$

Add these equations

$$3x = 5$$
$$x = \frac{5}{3}$$

substituting this value of x into second equation above

$$\frac{5}{3} + y = 2$$
$$y = 2 - \frac{5}{3}$$
$$= \frac{1}{3}$$

Question 8

(a) draw next pattern

(b)

Dots	1	2	3	4	5	6
Lines	4	7	10	13	16	19

(c)

$$99 \times 3 + 1 = 298$$

(d)

$$3n + 1$$

(e)

$$3n + 1 = 85$$

$$3n = 84$$

$$n = 28$$

There are 85 lines in the pattern with 28 dots

Question 9

(a)

$$\frac{360}{7} = 51.4 \text{ to 3 sig figs}$$

(b) (i) Isosceles

(ii) p=50
(180-30) q=80
ABC is isosceles, so 180 - (2 × 50) r=50
isosceles triangle, so same as p s=50
same as p t=80 *same as q*

(c)

$$y = 25$$

angle JKL is a right angle, being subtended by a diameter

Paper - 0580/04 - 0581/04 , October/November 2005

Question 1

(a)

$$800 \times 1.52$$

$$= 1216 \; Euros$$

(b)

$$\frac{173.46}{118} = 1.47 \; Euros$$

(c) The toy is 1.75 Euros less in Scotland

As a percentage

$$\frac{1.75}{11.50} \times 100 = 15.2 \text{ to 3 sig figs}$$

(d) Let x= cost for 1 child

$$2(2x) + 3x = 4347$$

$$4x + 3x = 4347$$

$$7x = 4347$$

$$x = 621$$

(e) Original cost

$$= \frac{4347}{90} \times 100$$

$$= 48.3 \times 100$$

$$= 4830 \; Euros$$

(f) (i)

$$\frac{2350}{3.25}$$

$$= 723 \; km/h \quad \text{to 3 sig figs}$$

(ii)

$$= \frac{723 \times 1000}{60 \times 60}$$

$$= 200.833....$$

$$= 201 \; m/s \text{ to 3 sig figs}$$

Question 2

(a) draw axes

(b) draw triangle

(c) (i) Add

$$\begin{bmatrix} 3 \\ -9 \end{bmatrix}$$

to each coordinate, producing

$$A_1(5, -7), \quad B_1(8, -7), \quad C_1(8, -5)$$

(ii) treat $x = -1$ as a 'mirror', and plot each vertex of image triangle at distances from this 'mirror' equal to the distance that the corresponding vertex of the original triangle is in 'front' of it, producing

$$A_2(-4, 2), \quad B_2(-7, 2), \quad C_2(-7, 4)$$

(iii) *Method in general for rotations would be :- with compasses on origin, mark off an arc through each vertex of the triangle. Along each arc measure off angle of rotation and mark new vertex (this angle of rotation is measured in a counter-clockwise direction).*

Because the rotation here is $180°$ *about the origin, the procedure is simpler - just produce negatives of all coordinates, producing*

$$A_3(-2, -2), \; B_3(-5, -2), \; C_3(-5, -4)$$

(d) (i)

$$\begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 5 \\ 2 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 7.5 & 7.5 \\ 2 & 2 & 4 \end{bmatrix}$$

producing a triangle with vertices

$$A_4(3, 2), \; B_4(7.5, 2), \; C_4(7.5, 4)$$

(ii) *since the inverse of*

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is

$$\frac{1}{|\mathbf{A}|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

required inverse is

$$\frac{1}{1.5} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{1.5} & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & 0 \\ 0 & 1 \end{bmatrix}$$

(iii) This produces a stretch with x-coordinates reduced by a factor of $\frac{2}{3}$ *and with y-coordinates unchanged.*

Question 3

(a) (i)

$$60°$$

(ii)

$$RS^2 = 7^2 + 15^2 - 2(7)(15)\cos 60$$

$$= 49 + 225 - 210\left(\frac{1}{2}\right)$$

$$= 274 - 105$$

$$= 169$$

so

$$SR = 13$$

(b) (i)

$$145°$$

(ii)

$$\frac{\sin PQR}{15} = \frac{\sin 55}{14}$$
$$\sin PQR = \frac{15 \sin 55}{14}$$
$$\angle PQR = \tan^{-1} \frac{15 \sin 55}{14}$$
$$\angle PQR = 61.361......$$
$$= 61.4^\circ \text{ to 3 sig figs}$$

(iii) now

$$\angle PRQ = 180 - (55 + 61.4) = 63.6$$
$$\frac{PQ}{\sin 63.6} = \frac{14}{\sin 55}$$
$$PQ = \frac{14 \sin 63.6}{\sin 55}$$
$$= 15.3km$$

(c) Area of $\triangle PSR$

$$= \frac{1}{2}(15)(7) \sin 60$$
$$= 45.4663...$$

Area of $\triangle PQR$

$$= \frac{1}{2}(15)(15.3) \sin 55$$
$$= 93.9976...$$

So, total area of forest

$$= 139.4639.........$$
$$= 139km^2 \quad \text{to 3 sig figs}$$

Question 4

(a) (i)

$$12$$

(ii)

$$3$$

(iii)

$$21$$

add up the numbers inside the 'bubbles'

(iv)

$$2$$

that section of B which does not intersect F

(v)

$$\frac{14}{24} = \frac{7}{12}$$

(vi)

$$\frac{12}{19}$$

(b) (i)

$$P(G) = \frac{12}{22} \times \frac{11}{21}$$
$$= \frac{2}{7}$$

(ii)

$$P[(BG)\ OR\ (GB)] = \left(\frac{10}{22} \times \frac{12}{21}\right) + \left(\frac{12}{22} \times \frac{10}{21}\right)$$
$$= 2\left(\frac{20}{77}\right)$$
$$= \frac{40}{77}$$

Question 5

(a) Missing values

$$x = 3 \Rightarrow f(x) = \frac{8}{9}$$

and the same value will be produced for $x = -3$

$$x = 0.3 \Rightarrow f(x) = 1 - \frac{1}{\frac{9}{100}}$$
$$= 1 - \frac{100}{9}$$
$$= -10\frac{1}{9}$$

and the same value will be produced for $x = -0.3$

(b) (i) draw axes

(ii) draw graph, symmetric about y-axis, but not crossing the x-axis (nothing drawn between -0.3 and $+0.3$)

(c) any integer ≥ 1.

(d) Draw straight line, passing through y = -5 and having a gradient of 2 (this time the line is continous, passing across the y-axis)

(e) (i) *need x-values where the two graphs intersect*

$$x = -0.4, \quad x = 0.4, \quad x = 2.9$$

or similar

(ii)

$$1 - \frac{1}{x^2} = 2x - 5$$
$$x^2 - 1 = 2x^3 - 5x^2$$
$$2x^3 - 6x^2 + 1$$

(f) (i) *could use a set square and straight edge to produce the required parallel line*

(ii)

$$y = 2x - 2$$

Question 6

(a)

$$2$$

in each case the plane includes P and mid-points of base lines

(b)

$$\frac{1}{3}(6 \times 5) \times 3$$

$$= 30cm^3$$

(c) *construct a triangle with vertices P, M and the mid-point of DA, height 3 and base of 6*

$$\tan\theta = \frac{3}{3} = 1 \Rightarrow \theta = 45^\circ$$

(d) *construct triangle PBD with height 3*

$$DB^2 = 5^2 + 6^2$$

$$= 25 + 36$$

$$= 61$$

so

$$DB = \sqrt{61}$$

therefore

$$\tan PBD = \frac{3}{\frac{\sqrt{61}}{2}}$$

$$= \frac{6}{\sqrt{61}}$$

so

$$\angle PBD = \tan^{-1}\frac{6}{\sqrt{61}}$$

$$= 37.5^\circ$$

so

(e)

$$PB^2 = 3^2 + \left(\frac{\sqrt{61}}{2}\right)^2$$

$$= 9 + \frac{61}{4}$$

$$= 24.25$$

so

$$PB = 4.92$$

Question 7

(a) (i)

$$35 \times \frac{1}{2}(2.5 + 1.1)$$

$$= 35 \times 1.8$$

$$= 63m^2$$

(ii)

$$63 \times 24 = 1512m^3$$

(iii)

$$1512 \times 1000 = 1,512,000 \;\; liters$$

(b) (i)

$$35.03 \times 24 \times 2.25 = \$1891.62$$

(ii)

$$\$1900$$

(c) (i)

$$\pi \times (12.5)^2 \times 14$$

$$= 6872.233.......cm^3$$

$$= 6872cm^3 \text{ to the nearest integer}$$

or

$$6870cm^3 \text{ to 3 sig figs}$$

(ii)

$$6870cm^3 = 0.006870m^3$$

Time to empty

$$= \frac{1512}{0.006870}$$

$$= 220087.33..\ sec$$

$$= 61.13\ hours$$

$$= 2\ days\ \ 13\ hours \text{ to required accuracy}$$

Question 8

(a) (i)

$$\frac{40}{x}$$

(ii)

$$\frac{40}{x} = \frac{40}{x+2} + 1$$

$$40(x+2) = 40x + (x)(x+2)$$

$$40x + 80 = 40x + x^2 + 2x$$

$$x^2 + 2x - 80 = 0$$

(iii)

$$x^2 + 2x - 80 = 0$$

$$(x - 8)(x + 10) = 0$$

so

$$x - 8 = 0 \Rightarrow x = 8$$

and

$$x + 10 = 0 \Rightarrow x = -10$$

(iv)

$$8$$

because this is the only realistic figure of the two solutions

(b) (i)

$$m = n + 2.55 \qquad (6)$$

$$2m = 5n \qquad (7)$$

(ii) (6) $\times 2$

$$2m = 2n + 5.10$$

subtracting this from (7)

$$0 = 3n - 5.1$$

$$3n = 5.1$$

$$n = 1.7$$

Question 9

(a)

$$160 < h \leq 170$$

(b) (i) Using the mid-class value as an estimate in each case, to sum the heights

$$(125 \times 15) + (135 \times 24) + (145 \times 36) + (155 \times 45) +$$

$$(165 \times 50) + (175 \times 43) + (185 \times 37) + (195 \times 20)$$

dividing this by 270 produces

$$162.33$$

(ii) This is an estimate precisely because the mid-class values were an estimate

(c) p = 15
q=39
r = 75

(d) Draw axes.

Insert 9 points at

$$(120, 0), (130, 15), (140, 39), (150, 75), (160, 120),$$

$$(170, 170), (180, 213), (190, 250), (200, 270)$$

(e) (i)

$$162$$

or similar, corresponding to 135 on the vertical axis

(ii)

$$176$$

or similar, corresponding to 203 on the vertical axis

(iii) the lower quartile

$$= 148$$

or similar, corresponding to 67.5 (or 68) on the vertical axis

Thus the inter-quartile range for these given values will be

$$176 - 148 = 28$$

(iv)

$$168$$

or similar, corresponding to 162 on the vertical scale

(f)

$$188cm$$

or similar, corresponding to 240 on the vertical scale