Cambridge **IGCSE Mathematics**

2004

Model Answers

Note the instructions ask you to give answers to 3 sig figs, where appropriate. (In general, the number of significant figures in an answer should not exceed the number of significant figures in the input data, or if this data has differing numbers of significant figures, the data with the lowest number of significant figures).

Brian Daugherty

Statements in italics are for information rather than a part of the answer

Paper 1 - 0580/01 - 0581/01 , May/June 2004

Question 1

$$4^3 - 5^2$$

$$= 64 - 25$$

$$= 39$$

Question 2

$$447 - (-395)$$

$$447 + 395$$

$$= 842m$$

Question 3

(a)

$$75\% = \frac{75}{100} = \frac{3}{4}$$

(b)

$$0.07 = \frac{7}{100}$$

Question 4

(a)

$$49$$

(b)

$$31$$

Question 5

Actual length

$$= 18 \times 25 = 450cm = 4.5m$$

Question 6

$$2\frac{1}{4} \div \frac{1}{2}$$

$$= \frac{9}{4} \times \frac{2}{1}$$

$$= \frac{18}{4}$$

$$= \frac{9}{2}$$

Question 7

$$141.5cm \leq height < 142.5cm$$

Question 8

$$4xy - 6xz$$

$$= 2x(2y - 3z)$$

Question 9

$$\frac{200}{1.05} = 190.476.... = 190.48\ euros \text{ to nearest cent}$$

(from the given exchange information, when you hand over euros are you going to get a larger or smaller no. of dollars? - that will tell you whether to multiply by 1.05 or divide by 1.05)

Question 10

(a)

$$0$$

(a line of symmetry acts like a mirror. If you have a proposal for a line of symmetry, cover up one side of the line, imagine that the line is a mirror and imagine whether this mirror will re-create the figure you are examining)

(b)

$$2$$

Question 11

Sinced $\triangle OPQ$ is isosceles

$$\angle POQ = 180 - (2 \times 35) = 110^\circ$$

Question 12

(a)

$$\left(\frac{1}{2}\right)^x = \frac{1}{8} \Rightarrow x = 3$$

(b)

$$7^y = 1 \Rightarrow y = 0$$

Question 13

(a) (i)

$$200 \div 40$$

(ii)

$$5$$

(b)

$$5.58....$$

$$= 5.6 \text{ to 2 sig figs}$$

(a useful technique here is being practised here - if you have a 'feel' for the numbers and therefore an idea of the general magnitude of the final answer, you can avoid the common error of being out by a factor of 10 (or 100 etc))

Question 14

To work out cost per one milliliter

Size A

$$\frac{130}{800} = 0.1625$$

Size B

$$\frac{230}{1500} = 0.15\dot{3}$$

So size B is cheapest

Question 15

$$\sin 32 = \frac{CB}{5}$$

$$CB = 5 \sin 32$$

$$CB = 2.649.... = 2.65 \text{ to 3 sig figs}$$

Question 16

(a)

$$y = a + bc$$

$$y = (-3) + (2)(8)$$

$$= -3 + 16$$

$$= 13$$

(b)

$$y = a + bc$$

$$a + bc = y$$

$$bc = y - a$$

$$c = \frac{y - a}{b}$$

Question 17

Construct bar chart - have bars of equal width and with height corresponding to figures in the table. Could have every square representing 5 vertically.

Question 18

(a) Money brought in

$$= 50 \times 0.25 = \$12.50$$

Profit

$$= 12.50 - 8 = \$4.50$$

(b) As a fraction

$$\frac{4.5}{8}$$

As a percentage

$$\frac{4.5}{8} \times 100 = 56.25\%$$

Question 19

(a) Area

$$= \pi(30)^2 = 2827.43..... = 2830cm^2 \text{ to 3 sig figs}$$

(instructions at the front of the paper request you to state an answer like this to 3 sig figs)

(b) Volume

$$= \pi(30)^2 \times 80cm^2$$

$$= \frac{\pi(30)^2 \times 80}{1000} \, litres$$

$$= 226.19.... = 226 \, litres \text{ to 3 sig figs}$$

Question 20

(a)

$$4x - 5 = 31$$

$$4x = 36$$

$$x = 9$$

(b)

$$4(y - 5) = 36$$

$$4y - 20 = 36$$

$$4y = 56$$

$$y = 14$$

Question 21

(a)

$$00:15 \text{ Monday}$$

(b) (i) From (a), aircraft leaves at 00:15 Dubai time.

So journey lasts from

$$00:15 - 07:45$$

$$= 7h \ 30m$$

(ii) Average Speed

$$= \frac{5620}{7.5} = 749.\dot{3} = 749km/h \text{ to 3 sig figs}$$

Paper 2 - 0580/02 - 0581/02, May/June 2004

Question 1

$$23:20(18th)\ to\ 02:40(19th)$$

takes

$$3h\ \ 20m$$

Question 2

$$10.87... = 10.9 \text{ to 3 sig figs}$$

Question 3

$$0.5^3,\ \ 0.5^2,\ \ \sqrt{0.5}$$

Question 4

$$\frac{2}{3}p^{12} \times \frac{3}{4}p^8$$

$$= \frac{6}{12}p^{20}$$

$$= \frac{1}{2}p^{20}$$

Question 5

$$\frac{x}{4} - 8 = -2$$

$$x - 32 = -8$$

$$x = 24$$

Question 6

$$6375 \leq P < 6385$$

Question 7

$$\frac{-\frac{1}{2} - \frac{3}{8}}{-\frac{1}{2} + \frac{3}{8}} = \frac{-\frac{4}{8} - \frac{3}{8}}{-\frac{4}{8} + \frac{3}{8}} = \frac{-\frac{7}{8}}{-\frac{1}{8}} = 7$$

(If you are confident with the procedure, you could get 'rid' of all the denominators of 8 on the second line instead of the third)

Question 8

(a)

4

(b)

4

Question 9

Interest

$$= \frac{7.5}{100} \times 3000 = \$225$$

After two years the interest will be

$$225 \times 2 = \$450$$

(could also calculate the interest as 0.075×3000*)*

Question 10

(a)

$$80\ 000\ km^2$$

(b)

$$8.0 \times 10^4\ km^2$$

Question 11

$$\frac{1}{2}x + y = 5 \quad (1)$$

$$x - 2y = 6 \quad (2)$$

(1) × 2

$$x + 2y = 10 \quad (3)$$

Add (2) and (3)

$$2x = 16$$

$$x = 8$$

and from (2)

$$8 - 2y = 6$$

$$2y = 8 - 6 = 2$$

$$y = 1$$

Question 12

The sector for N Ireland has a central angle of $12°$*, so* $6°$ *corresponds to 1 million*

England 50
Scotland 5
Wales 3
N Ireland 2

Question 13

$$c = kd^2 + e$$

$$kd^2 + e = c$$

$$kd^2 = c - e$$

$$d^2 = \frac{c-e}{k}$$

$$d = \sqrt{\frac{c-e}{k}}$$

Question 14

(a) Placing compasses (with a width of 4cm) on P, draw an arc inside the garden

(b) Area

$$= \frac{1}{4} \times \pi(4)^2$$

$$= 4\pi = 12.56.. = 12.6 \text{ to 3 sig figs}$$

(top line is one quarter of the area of a circle)

Question 15

Since the triangles are similar, the ratio of the areas will equal the square of the scale factor

$$\frac{99}{11} = 9$$

So scale factor is 3

Thus

$$PQ = 3 \times BC$$

$$BC = \frac{PQ}{3} = \frac{12}{3} = 4cm$$

Question 16

(a)

$$\vec{OB} = \mathbf{a} + \mathbf{c}$$

(b)

$$\vec{CA} = \mathbf{a} - \mathbf{c}$$

(c) using (b)

$$\vec{OM} = \mathbf{c} + \frac{1}{2}(\mathbf{a} - \mathbf{c})$$

$$= \mathbf{c} + \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{c}$$

$$= \frac{1}{2}(\mathbf{a} + \mathbf{c})$$

now

$$\vec{BM} = \vec{OM} - \vec{OB}$$

$$= \frac{1}{2}(\mathbf{a} + \mathbf{c}) - (\mathbf{a} + \mathbf{c})$$

$$= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c} - \mathbf{a} - \mathbf{c}$$

$$= -\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{c}$$

$$= -\frac{1}{2}(\mathbf{a} + \mathbf{c})$$

Question 17

(a) (i) Join BD. With compasses on B and D in turn, draw arcs above and below BD such that they intersect. Use straight edge to join these intersections.

(ii) With compasses on D, draw arcs intersecting DC and DA. With compasses on each of these intersects in turn, construct arcs that themselves intersect with each other. Join D to this latter intersection.

(b) Use bisectors and choose area that is both to the 'right' of the bisector of BD and to the right of bisector of angle ADC.

Question 18

(a) BR is 'side' between Bruges and Rotterdam.

Using Pythagoras

$$BR^2 = 78^2 + 83^2$$

$$BR = 113.89.. = 114km \text{ to 3 sig figs}$$

(b) Bearing is identical; to angle at Rotterdam (alternate angles), so

$$\tan B = \frac{83}{78}$$

so B(bearing)

$$= \tan^{-1}\frac{83}{78} = 46.778... = 47^\circ \text{ to nearest degree}$$

Question 19

(a)

$$gf(x) = 2\left(\frac{x+1}{2}\right) + 1$$

$$= x + 1 + 1$$

$$= x + 2$$

so

$$gf(9) = 9 + 2 = 11$$

(b) given already in (a), i.e. $gf(x) = x + 2$

(c)

$$y = 2x + 1$$

$$2x + 1 = y$$

$$2x = y - 1$$

$$x = \frac{y-1}{2}$$

This $g^{-1}(y)$, but by just swopping the parameter *(which we can do at will because the parameter is only a dummy parameter - we can use any letter to represent it)*

$$g^{-1}(x) = \frac{x-1}{2}$$

Question 20

(a)

$$12x^2 - 3y^2$$

$$3(4x^2 - y^2)$$

(b) (i)

$$(x-3)^2 = (x-3)(x-3)$$
$$= x^2 - 3x - 3x + 9$$
$$= x^2 - 6x + 9$$

(ii)

$$x^2 - 6x + 10$$

Completing the square

$$(x-3)^2 - 9 + 10$$
$$= (x-3)^2 + 1$$

Sp $p = 3,\ q = 1$

Question 21

(a) Acceleration

$$= \frac{18}{10} = 1.8m/s^2$$

(ie the gradient of the line between 0-10 secs)

(b) Distance travelled in the first 30 secs equals the area under the graph to 30 secs

$$= \frac{1}{2}(10)(18) + (20 \times 18)$$
$$= 90 + 360$$
$$= 450m$$

(c) Between 30 and 45 secs, the distance travelled

$$= \frac{1}{2}(15)(18) = 135m$$

So total distance travelled

$$= 450 + 135 = 585m$$

So average speed

$$= \frac{585}{45} = 13m/s$$

Question 22

(a) (ii) BA *(the no of columns of B is not equal to no. of rows of A)*

(b)

$$\mathbf{BC} = \begin{bmatrix} 2 & 6 \\ 5 & -4 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 38 & 0 \\ 0 & 38 \end{bmatrix}$$

(c)

$$\mathbf{B}^{-1} = \frac{1}{38} \begin{bmatrix} 4 & 6 \\ 5 & -2 \end{bmatrix}$$

(since $\mathbf{BB}^{-1} = \mathbf{I}$*)*

Paper 3 - 0580/03 - 0581/03, May/June 2004

Question 1

(a) (i)

$$51$$

(ii) data in order is

$$12, 34, 35, 39, 48, 50, 51, 51, 65, 75$$

Median

$$= \frac{1}{2}(48 + 50) = 49$$

(iii) Mean

$$= \frac{460}{10} = 46$$

(b) (i) There are 90 students, so 1 student corresponds to an angle of 4°

So required angles are

A : 20
B: 60
C: 160
D: 80
E: 40

(ii) Draw pie chart using these angles above

(iii) (a)

$$\frac{40}{90} = \frac{4}{9}$$

(b)

$$\frac{20}{90} + \frac{10}{90} = \frac{30}{90} = \frac{1}{3}$$

Question 2

(a) Area of roof

$$= 3 \times 3 = 9m^2$$

(b) (i) Area of one wall

$$= 3 \times 2 = 6m^2$$

(ii) Area of 3 walls

$$= 6 \times 3 = 18m^2$$

(c) (i)

$$3 \times 0.2 = 0.6m^2$$

(ii) No of pieces of wood

$$= \frac{18}{0.6} = 30$$

(d) Volume

$$= 2 \times 0.1 \times 0.1 = 0.02m^2$$

(e)

Posts : 4.80
Wood : 9.00
Roof : 14.40
Nails : 2.10

Total 30.30

Question 3

(a)

$$y = 8 - x^2$$

Completing the table

$x = -1 \quad \rightarrow y = 8 - (-1)^2 = 8 - 1 = 7$
$x = 0 \quad \rightarrow y = 8$
$x = 2 \quad \rightarrow y = 8 - 2^2 = 8 - 4 = 4$
$x = 3 \quad \rightarrow y = 8 - 3^2 = 8 - 9 = -1$

(b) Draw graph with data from (a) - parabola, symmetric about the y-axis with a maximum at $y = 8$

(c) Answers in the region of

$$\pm 2.8$$

(d)

$$y = 2x + 5$$

Completing table

$x = -3 \quad \rightarrow y = 2(-3) + 5 = -6 + 5 = -1$
$x = 0 \quad \rightarrow y = 5$

(e) Draw line for data above. A straight line, intersecting the y-axis at $y = 5$ and with gradient 2

(f)

$$2$$

(i.e. m in $y = mx + c$)

(g)

$$x = -3, \;\; x = 1$$

Question 4

(a)

$$s = 360 - (80 + 50 + 110) = 120^\circ$$

(b)

$$3t = 360 - 2(75)$$

$$3t = 120$$

$$t = 70^\circ$$

(c) (i)

$$x + 2y = 180 - 50$$

$$x + 2y = 130$$

(ii)

$$x + y = 360 - 260 = 100$$

(iii) Eqn from (i) less the eqn from (ii) produces

$$y = 30$$

Into the eqn from (ii)

$$x + 30 = 100$$

$$x = 70$$

Question 5

(a)

$$200m \rightarrow 0.2km$$

(b) (i) The angle between a tangent to a circle and the corresponding radius line is a right angle

(ii)

$$PC = 7.8 + 0.2$$

$$= 8km$$

(iii) Using Pythagoras on $\triangle PSC$

$$7.8^2 + PS^2 = 8^2$$

$$PS^2 = 8^2 - 7.8^2$$

so

$$PS = 3.16km$$

(iv) Area

$$= \frac{1}{2}(7.8)(3.16)$$

$$= 12.324$$

$$= 12km^2 \text{ to 2 sig figs}$$

Question 6

(a) (i) translation given by

$$\begin{bmatrix} 10 \\ -2 \end{bmatrix}$$

(ii) Rotation about origin counter-clockwise by

$$90^\circ$$

(b) (i) Treat the y-axis as a 'mirror' to draw the reflection of E. All you need to do is make the x-coordinates positive, to produce a figure with the following coordinates

$$(5,3),(8,3),(6,1),(6,2),(5,2),(8,2),(7,1),(7,2)$$

(ii) *(when the center of enlargement is the origin, we just need to multiply all the coordinates of the vertices by the scale factor)*

F maps to a triangle with vertices

$$(2,4),\ (4,4),\ (2,8)$$

Question 7

(a) (i) Pentagon

(ii) Sum of interior angles

$$= (n-2) \times 180$$

$$= 3 \times 180$$

$$= 540^\circ$$

(iii)

$$\frac{540}{5} = 108^\circ$$

(b) (i) Using alternate angles

$$x = 70^\circ$$

also

$$y = 180 - 160 = 20^\circ$$

so

$$x + y = 70 + 20 = 90^\circ$$

(ii)

$$\tan BAC = \frac{120}{100} = 1.2$$

so

$$\angle BAC = 50.1944.. = 50.2 \text{ to 1 dp}$$

(iii) Bearing

$$= 70 + 50.2$$

$$= 120^\circ \text{ to 3 sig figs}$$

(iv)

$$180 + 120^\circ = 300^\circ \text{ to 3 sig figs}$$

(making use of alternate angles)

Question 8

Print-outs of the exam paper might not be exactly the same size as the original exam paper, so distances could be a bit distorted

(a) (i)

$$6cm$$

(ii)

$$10km$$

(iii)

$$75km$$

(b) Draw a line 1cm parallel to road, on both sides of road. This is best done using a set square which is placed with one edge along the road, another edge being placed on a straight edge (ruler), allowing it to slide along parallel to the road for a distance of 1cm.

(c) With compasses on A and B in turn, construct two arcs from each point that intersect 'above' and 'below' the points. Join these intersections up.

(d) Draw a circle of 4cms centered on C

(e) The airport is at the intersection of the bisector of AB and a line 1cm from the road. The circle from (d) allows to choose which point, i.e. the one 'below' the road.

Question 9

(a) (i)

$$12$$

(ii) General formula is

$$2n + 2$$

so if $n = 9$

$$2n + 2 = 20$$

(iii) as above

$$2n + 2$$

(b) (i) (a) General formula is

$$4n$$

where n is the no. of the diagram
so for diagram 5

$$4 \times 5 = 20 \; dots$$

(b) General formula here is

$$n^2$$

so for diagram 5

$$5^2 = 25 \; squares$$

(ii)

$$144 \rightarrow 12^2 \Rightarrow n = 12$$

no of dots for $n = 12$

$$= 4 \times 12 = 48$$

(iii)

$$4n = 40 \Rightarrow n = 10$$

No of squares for $n = 10$

$$= 10^2 = 100$$

Paper 4 - 0580/04 - 0581/04, May/June 2004

Question 1

(a) (i) Fatima pays

$$\frac{60}{100} \times 120 + (6 \times 10)$$

$$= 72 + 60$$

$$= \$132$$

(ii)

$$\frac{132}{120} \times 100 = 110\%$$

(i.e. state as a fraction and then multiply by 100)

(b) \$159.10 represents 86%, so original price

$$= \frac{159.10}{86} \times 100 = \$185$$

(c) Frame Size

$$\frac{156}{169} \times 52 = 48cm$$

(d) (i) Fatima rides

$$\frac{11}{20} \times 36 = 19.8cm$$

(ii) The total distance

$$= \frac{36}{2} \times 23 = 414km$$

Question 2

(a) (i)

$$f(x) = x^2 - x - 3$$

$p = (-3)^2 - (-3) - 3 = 9 + 3 - 3 = 9$
$q = 1^2 - 1 - 3 = -3$
$r = 4^2 - 4 - 3 = 9$

(ii) Draw graph using given data. Parabola with minimum between $(0, -3)$ and $(1, -3)$

(iii) Estimate a line representing gradient at $x = -1$. Extend this line and calculate its gradient -expect answer in region of -3

(b) (i)

$$g(x) = 6 - \frac{x^3}{3}$$

$$u = 6 - \frac{(-1)^3}{3} = 6 - \left(-\frac{1}{3}\right) = 6\frac{1}{3}$$

$$v = 6$$

(ii) Draw graph using given data

(c) (i)

$$x^2 - x - 3 = 6 - \frac{x^3}{3}$$

$$3x^2 - 3x - 9 = 18 - x^3$$

$$x^3 + 3x^2 - 3x - 27 = 0$$

(ii) Need the points where the two lines intersect.

Something of the order of

$$(2.5, 0.6)$$

Question 3

(a) (i)

$$36cm$$

(the nearest whole number corresponding to 182/183)

(ii)

$$26 - 46cm \quad (20cm)$$

(iii)

$$\approx 33.25cm$$

number corresponding to ≈ 145

(iv)

$$280$$

85 below 25cm, so ($365 - 85$*) above 25cm*

(b) (i) $q = 15$ from graph.

$$p = 365 - (17 + 41 + 62 + 98 + 85 + 15)$$

$$= 365 - 318 = 47$$

(ii)

Depth	mid pt	No. days	m.p. x days
$0 < d \leq 10$	5	17	85
$10 < d \leq 20$	15	41	615
$20 < d \leq 30$	25	62	1550
$30 < d \leq 40$	35	98	3430
$40 < d \leq 50$	45	85	3825
$50 < d \leq 60$	55	47	2585
$60 < d \leq 70$	65	15	975

Total

$$= \frac{13065}{365} = 35.8..\text{ to 3 sig figs}$$

(c) No of squares occupied by second column

$$= 20 \times 8 = 160$$

First column has the same width as the second column, so height

$$= \frac{8}{160} \times 58 = 2.9cm$$

Third column is 1.5 times as wide, so its height

$$= \frac{2}{3} \times \frac{8}{160} \times 147 = 4.9cm$$

Question 4

(a) Using Cosine Rule

$$AC^2 = 11.1^2 + 9.5^2 - 2(11.1)(9.5)\cos 70$$
$$= 141.3279..$$

so

$$AC = 11.88... = 11.9 \text{ to 3 sig figs}$$

(b) In cyclic quadrilaterals, opposite angles sum to 360°, so

$$\angle ADC = 180 - \angle ABC$$

(c)

$$\angle ACD = 180 - (110 + 37) = 33°$$

Using Sine Rule

$$\frac{11.9}{\sin 110} = \frac{AD}{\sin 33}$$
$$AD = \frac{11.9 \sin 33}{\sin 110}$$
$$= 6.89715...$$
$$= 6.90 \text{ to 3 sig figs}$$

(d) (i)

$$70°$$

(angle subtended by AC at circumference)

(ii) To find CE

$$\angle EAC = \frac{1}{2}(180 - 70) = 55°$$

so

$$\frac{CE}{\sin 55} = \frac{11.9}{\sin 70}$$
$$CE = \frac{11.9 \sin 55}{\sin 70}$$

Area of ACE

$$= \frac{1}{2}(11.9)(CE)\sin 55$$
$$= \frac{1}{2} \times \frac{11.9^2 \times \sin^2 55}{\sin 70}$$
$$= 50.560.... = 50.6cm^2 \text{ to 3 sig figs}$$

Question 5

(a) Time taken

$$= \frac{10}{x}$$

(b)

$$\frac{10}{x} - \frac{10}{x+1} = \frac{1}{2}$$
$$(x+1)(10) - x(10) = x(x+1)(\frac{1}{2})$$
$$10x + 10 - 10x = \frac{1}{2}(x^2 + x)$$
$$20x + 20 - 20x = x^2 + x$$
$$20 = x^2 + x$$
$$x^2 + x - 20 = 0$$

(c)

$$x^2 + x - 20 = 0$$
$$(x+5)(x-4) = 0$$

so

$$x + 5 = 0 \Rightarrow x = -5$$

or

$$x - 4 = 0 \Rightarrow x = 4$$

(d) Only physical solution from (c) is $x = 4$, so time to waterfall

$$= \frac{10}{4} = 2.5h$$

Question 6

(a) (i) Volume of cone

$$= \frac{1}{3}\pi(7)^2(13)$$

Volume of hemisphere

$$= \frac{2}{3}\pi(7)^3$$

Total

$$= \frac{1}{3} \times 7^2 \times \pi(13 + 2 \times 7)$$
$$= \frac{49\pi}{3}(27)$$
$$= 49\pi(9)$$
$$= 441\pi$$
$$= 1385.4423... = 1385.4cm^2 \text{ to 1 dp}$$

(ii) Mass

$$= 1385.4423... \times 0.94$$
$$= 1302.3158...g$$
$$= 1.3kg \text{ to 1 dp}$$

(b) Length of sloping edge l is given by

$$l^2 = 13^2 + 7^2$$
$$= 169 + 49$$
$$= 218$$

so

$$l = \sqrt{218}$$

Surface Area

$$= \pi(7)\sqrt{218}$$
$$= 324.6954.... = 325cm^2 \text{ to 3 sig figs}$$

(c) Area of hemisphere

$$= 2\pi r^2$$
$$= 2\pi(7)^2$$
$$= 98\pi$$

Cost of gold plate per cm^2

$$= \frac{411.58}{324.6954.... + 98\pi}$$
$$= 0.65064.... = \$0.65 \text{ to nearest whole cent}$$

Question 7

(a) (i) Complete Venn Diagram - 3 are outside the circles and 8 in the intersection, leaving 12 studying Physics only and 7 studying Chemistry only.

(ii)

$$8$$

(as already mentioned in (i)

(iii)

$$\frac{12}{30}$$

(iv)

$$\frac{12}{20} = \frac{3}{5}$$

(b) (i)

$$P(BB) = \frac{3}{9} \times \frac{4}{10} = \frac{12}{90} = \frac{2}{15}$$

(ii)

$$P(WW) + P(WB) + P(BW)$$

$$= \left(\frac{6}{9} \times \frac{6}{10}\right) + \left(\frac{6}{9} \times \frac{4}{10}\right) + \left(\frac{3}{9} \times \frac{6}{10}\right)$$

$$= \frac{36}{90} + \frac{24}{90} + \frac{18}{90}$$

$$= \frac{78}{90} = \frac{13}{15}$$

(iii)

$$P(WWWW)$$

$$= \left(\frac{6}{9} \times \frac{6}{10} \times \frac{5}{8} \times \frac{5}{9}\right)$$

$$= \frac{5}{36}$$

(iv)

$$P(BBBB) = \left(\frac{3}{9} \times \frac{4}{10} \times \frac{2}{8} \times \frac{3}{9}\right)$$

$$= \frac{1}{90}$$

Probability not all same color

$$= 1 - \frac{5}{36} - \frac{1}{90}$$

$$= 1 - \frac{25-2}{180}$$

$$= 1 - \frac{27}{180}$$

$$= \frac{153}{180} = \frac{17}{20}$$

Question 8

(a) (i) Anti-Clockwise (or positive) rotation of 90° about the origin

(ii) translation described by

$$\begin{bmatrix} -2 \\ -5 \end{bmatrix}$$

(iii) Reflection about the line $y = -x$

(iv) Clockwise (or negative) rotation of 180° about $(1, -1)$

(v) Enlargement by scale factor of 2 about origin

(vi) Shear in the y-direction, x unchanged

(b) Applying the transformation to the vertices of A.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 & -2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$$

and the resultant coordinates belong to B.

(c) (i) B onto D is a reflection about the x-axis. The required matrix is therefore

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Question 9

(a)

$$15x + 25y \leq 2000$$

ie

$$3x + 5y \leq 400$$

(b)

$$x \geq y$$

(c)

$$y \geq 35$$

(d) (i) set up coordinates

(ii) draw three lines mentioned, i.e. in each case, replace the inequality sign by an equal sign and draw the resultant straight line. The area then satisfying each inequality is to one side of this straight line - which side to choose can be determined by just choosing one point and testing this point to see whether it satisfies the inequality or not.

You are asked to shade the unwanted area, which is the area outside a triangle which vertices

$$(35, 35),\ (74, 35),\ (50, 50)$$

(e)

$$3x + 5y \leq 400$$

If 70 pencils are bought

$$210 + 5y \leq 400$$

$$5y \leq 190$$

$$y \leq 38$$

So largest possible number of pens is 38

(f) The largest possible profit will occur at a point represented by a vertex of the triangle mentioned in (d)(ii)

The expression for profit (P)

$$P = 5x + 7y$$

using the data at each of the three vertices in turn gives

$$P = 35(5) + 35(7) = 420$$

$$P = 74(5) + 35(7) = 615$$

$$P = 50(5) + 50(7) = 600$$

So the greatest possible profit is

$$615\ cents\ = \$6.15$$