Edexcel **GCSE Mathematics (1387)**

Higher Tier 2004

Model Answers

In general, the number of significant figures in an answer should not exceed the number of significant figures in the input data, or if this data has differing numbers of significant figures, the data with the lowest number of significant figures.

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Statements in italics are for information rather than a part of the answer

Paper 5 (Non-Calculator), 8 June 2004

Question 1

(a) (i)

$$2.21$$

(ii)

$$0.013$$

(b)

$$39 = 3 \times 13$$

$$17 \text{ is already prime}$$

so

$$\text{LCM} = 3 \times 13 \times 17$$
$$= 3 \times 221 = 663$$

Question 2

Three required expressions are

$$\frac{\pi abc}{2d}, 2a^2, 2(c^2 + d^2)$$

Question 3

(a)

$$0.2 \times 200 = 40$$

(b) $\text{P}(4 \text{ or } 6) = \text{P}(4) + \text{P}(6)$

$$= 0.2 + 0.4 = 0.6$$

Question 4

(a)

$$108 = 2 \times 54$$
$$= 2^2 \times 27$$
$$= 2^2 \times 3 \times 9$$

(b)

$$24 = 2 \times 12$$
$$= 2^2 \times 6$$
$$= 2^3 \times 3$$

So

$$\text{H.C.F.} = 2^2 \times 3 = 12$$

Question 5

Set compasses to length PB and mark off an equal distance the other side of P (which I will call C). Using a slightly wider distance place point of compasses on B and mark off arcs above and below P. Place compasses on C and repeat operation. Connect the two points where these arcs intersect, to form required perpendicular.

Question 6

$$\text{Volume} = 15 \times 10 = 150cm^3$$

Question 7

(a)

$$k^5 \div k^2 = k^3$$

(b) (i)

$$4(x + 5) + 3(x - 7)$$
$$= 4x + 20 + 3x - 21$$
$$= 7x - 1$$

(ii)

$$(x + 3y)(x + 2y)$$
$$= x^2 + 2xy + 3xy + 6y^2$$
$$= x^2 + 5xy + 6y^2$$

(c)

$$(p + q)^2 + 5(p + q)$$
$$= (p + q)(p + q + 5)$$

(d)

$$\left(m^{-4}\right)^{-2}$$
$$m^8$$

(e)

$$2t^2 \times 3r^3t^4$$
$$6r^3t^6$$

Question 8

(i)

$$100.5mm$$

(ii)

$$101.5mm$$

Question 9

Area of triangle

$$= \frac{1}{2} \times \frac{5}{8} \times 6\frac{2}{5}$$

$$= \frac{1}{2} \times \frac{5}{8} \times \frac{32}{5}$$

$$= 2cm^2$$

Area of square

$$= 18 \times 2 = 36cm^2$$

Therefore the side of square

$$= 6cm$$

Therefore, perimeter

$$= 4 \times 6 = 24cm$$

Question 10

(a)

$$6y + 5x = 15$$

$$6y = -5x + 15$$

$$y = -\frac{5}{6}x + \frac{5}{2}$$

(b) When x = -21 and y = k

$$k = -\frac{5}{6}(-21) + \frac{5}{2}$$

$$= \frac{105}{6} + \frac{5}{2}$$

$$= 17.5 + 2.5$$

$$= 20$$

(c) (i) Lines enclose area bounded by both axes, $6y + 5x = 15$ (as shown on graph) and the line x=1.5

(ii)

$$(1, 1)$$

Question 11

(a)

$$y = 2x + 6$$

(b) Gradient of BC (m) is given by

$$m(2) = -1$$

$$m = -\frac{1}{2}$$

so equation of BC is

$$y = -\frac{1}{2}x + 6$$

(c) The rule is that if opposite angles of the quadilateral equal 180 degrees then it can be circumscribed. This condition is always satisfied for rectangles.

Question 12

Each apex of triangle transforms so

$$(0, 0) \rightarrow (2, 2)$$

$$(-4, 0) \rightarrow (-4, 2)$$

$$(-2, -4) \rightarrow (-1, -4)$$

Question 13

(a)

$$32s$$

(Note : Corresponding to 20 on vertical axis)

(b) Box plot of form as shown for (c), except the box itself stretches from 16 (the lower quartile) to 45 (the upper quartile), with center line of 32 (representing the median, corresponding to 20 on the vertical axis). Lines projecting either side extend to 9 on the left and 57 to the right.

(c) Apart from straightforward comparison of items mentioned in (b), could also comment on the fact that the inter-quartile range is less for the girls, as is the range.

Question 14

Time (t minutes)	Frequency
$0 < t \leq 10$	20
$10 < t \leq 15$	18
$15 < t \leq 30$	45
$30 < t \leq 50$	52

Question 15

(a) On Jan 1st 2000, t=0, V =1600

$$1600 = pq^0 \Rightarrow p = 1600$$

On Jan 1st 2002

$$400 = pq^2 \Rightarrow 400 = 1600q^2 \Rightarrow q^2 = \frac{1}{4} \Rightarrow q = \frac{1}{2}$$

(b)

$$V = 1600\left(\frac{1}{2}\right)^8$$

$$V = 1600 \times \frac{1}{256} \Rightarrow V = £6.25$$

Question 16

(a) Since $\angle ABC$ and $\angle ACB$ are equal, then $\triangle ABC$ is isosceles with $AB = AC$. Since PB and PC are tangents, both radiating from P, they are of equal length. Since AP is shared by both triangles, $\triangle APB$ and $\triangle APC$ are congruent.

(b) By symmetry, angle BPC = 20 degrees. Since triangle PBC is isosceles, angle PBC = 80 degrees.

Using alternate segment theorem, angle BAC = 80 degrees.

Since $\triangle$ ABC is isosceles, angle ABC = 50 degrees.

Question 17

(a)

$$\vec{AC} = 6\mathbf{c} - 6\mathbf{a} = 6(\mathbf{c} - \mathbf{a})$$

$$\vec{OP} = \vec{OA} + \frac{2}{3}\vec{AC}$$

$$\vec{OP} = 6\mathbf{a} + 4(\mathbf{c} - \mathbf{a})$$

$$= 6\mathbf{a} + 4\mathbf{c} - 4\mathbf{a}$$

$$= 2\mathbf{a} + 4\mathbf{c}$$

(b)

$$\vec{CB} = 6\mathbf{a}$$

$$\vec{OM} = \vec{OC} + \frac{1}{2}\vec{CB}$$

$$\vec{OM} = 6\mathbf{c} + \frac{1}{2}(6\mathbf{a})$$

$$= 6\mathbf{c} + 3\mathbf{a}$$

$$= \frac{3}{2}(2\mathbf{a} + 4\mathbf{c})$$

Therefore O,P and M lie on a straight line, since

$$\vec{OM} = \frac{3}{2}\vec{OP}$$

Question 18

(a)

$$16^{\frac{1}{2}} = 4$$

(b)

$$\sqrt{40} = \sqrt{4 \times 10} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$$

Therefore

$$k = 2$$

(c) The area of hole as a fraction of the area of the large rectangle

$$\frac{\sqrt{2}\sqrt{5}}{\sqrt{8}\left(\sqrt{5} + \sqrt{20}\right)}$$

$$= \frac{\sqrt{2}\sqrt{5}}{\sqrt{8}\sqrt{5} + \sqrt{8}\sqrt{20)}}$$

$$= \frac{\sqrt{2}\sqrt{5}}{\sqrt{2}\sqrt{4}\sqrt{5} + \sqrt{2}\sqrt{4}\sqrt{4}\sqrt{5)}}$$

$$= \frac{1}{\sqrt{4} + \sqrt{4}\sqrt{4)}}$$

$$= \frac{1}{6}$$

Therefore, area of card as fraction of large rectangle

$$= 1 - \frac{1}{6} = \frac{5}{6}$$

which as a percentage is

$$\frac{5}{6} \times 100 = 83\frac{1}{3}\%$$

Question 19

(a) (i)

$$2x^2 - 35x + 98$$

$$= (2x - 7)(x - 14)$$

(ii) Equation becomes

$$= (2x - 7)(x - 14) = 0$$

so

$$2x - 7 = 0 \Rightarrow x = \frac{3}{2}$$

or

$$x - 14 = 0 \Rightarrow \ x = 14$$

(b) (i)

$$\frac{7}{n + 7}$$

(ii) Assume that a probability of $\frac{2}{5}$ is possible

$$\frac{7}{n+7} = \frac{2}{5}$$

$$n + 7 = \frac{35}{2}$$

which implies that n is non-integer. Therefore original assumption is wrong

(c)

$$P(yellow) = \frac{n}{n+7}$$

P(different colors) = P(John/yellow)P(Mary/white) or P(Mary/white)P(John/yellow)

so

$$2\left(\frac{7}{n+7} \times \frac{n}{n+7}\right) = \frac{4}{9}$$

$$2\left(\frac{7n}{(n+7)^2}\right) = \frac{4}{9}$$

$$18(7n) = 4(n+7)^2$$

$$126n = 4(n^2 + 14n + 49)$$

$$4n^2 - 70n + 196 = 0$$

$$2n^2 - 35n + 98 = 0$$

(d) Solution of above eqn., from (a)(ii)

$n = 14$ (disregarding the other solution as unphysical)

so

$$P(white) = \frac{7}{n+7} = \frac{7}{21} = \frac{1}{3}$$

so

$$\text{P(Mary/white) and P(John/white)} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

Question 20

(a) (i)

$$y = \sin x^\circ + 1$$

(ii)

$$y = 2\sin x^\circ$$

(b) Increase the amplitude (i.e. the y value) by a factor of 3

Halve the wavelength (i.e. the x value)

Paper 6 (Calculator), 15 June 2004

(Note : When a number is given followed by several dots, this implies that the number is to be kept inside the calculator in full, and this full form is to be used for the entire calculation. You do not write it down in truncated form and then use this truncated form in later calcuulations - this will possibly introduce rounding errors.

Question 1

How much money, on average, do you spend on each visit to the canteen? (With relevant boxes, e.g. $£1, £1.50, £2, £2.50$

Question 2

Shaded circle of radius 3 cm, centered on Manchester

Question 3

(a)

$$Feb = \frac{147 + 161 + 238}{3} = 182$$

$$Mar = \frac{161 + 238 + 135}{3} = 178$$

$$Apr = \frac{238 + 135 + 167}{3} = 180$$

$$May = \frac{135 + 167 + 250}{3} = 184$$

(b) 20% off the normal prices gives 80% of normal price

Taking 30% off this 80% means a further deduction of

$$80 \times 0.3 = 24\% \text{ off normal prices}$$

giving a total deduction of 54% off normal prices

Question 4

x	$x^3 - 2x$
4	56
5	115
4.5	82.125
4.3	70.907
4.2	65.688

So answer is between 4.2 and 4.3

4.25	68.27

So answer is 4.2 to 1 d.p.

Question 5

(a)

$$10^{-9}$$

(b) No of calculations per second

$$= \frac{1}{5 \times 10^{-9}}$$

$$= 0.2 \times 10^9$$

$$= 2 \times 10^8$$

Question 6

(a)

1.9626315789

(b)

1.96

Question 7

	1	2	3	4
No. of dots	10	14	18	22

A constant first difference of 4 $\Rightarrow$ the general expression is

$$4n + k$$

where k is a constant. Inspection shows $k = 6$, so general term is

$$4n + 6$$

Question 8

Volume of the puck

$$= \pi(3.8)^2 \times 2.5 \ cm^3$$

Mass of the puck

$$= \pi(3.8)^2 \times 2.5 \times 1.5$$

$$= 170g$$

Question 9

(a) Using Pythagoras

$$DG^2 = 6^2 + 10^2 = 136$$

$$DG = 11.7m$$

(b)

$$\cos x = \frac{8}{10}$$

$$x = 36.9^\circ$$

Question 10

$$6x - 2y = 33 \quad (1)$$

$$4x + 3y = 9 \quad (2)$$

(1) $\times 2$

$$12x - 4y = 66 \quad (3)$$

(2) $\times 3$

$$12x + 9y = 27 \quad (4)$$

(4) - (3)

$$13y = -39$$

$$y = -13$$

Into (2)

$$4x + 3(-13) = 9$$

$$4x = 48$$

$$x = 12$$

Question 11

$$\frac{133}{72} = 1.847$$

$$\frac{160}{82} = 1.951$$

since ratios are different, shapes are not similar

Question 12

(a) After three years, value

$$= 12000(1 - 0.25)^3$$

$$= 12000(0.75)^3$$

$$= £5062.50$$

(b)

$$(1 - 0.2)^4 = (0.8)^4 = 0.4096$$

Question 13

The volume (V) of a cone is given by

$$V = \frac{1}{3}\pi r^2 h$$

so

$$r^2 = \frac{3V}{\pi h}$$

$$= \frac{3 \times 10}{\pi \times 1.5}$$

therefore

$$r = 2.5231....$$

to 3 significant figures

$$r = 2.52cm$$

Question 14

(a) Complete 'first choice' by inserting 0.4 at the bottom

The 'second choice' will be the same top and bottom, each of which is also identical with the 'first choice', i.e. 0.6 and 0.4 respectively.

(b)

$$0.6 \times 0.6 = 0.36$$

(c) Total playing time of the 5 CDs

$$= (30 \times 42) - (25 \times 42.8) = 190$$

So mean time of 5 CDs sold

$$= \frac{190}{5} = 38mins$$

Question 15

(a)

$$S \ \alpha \ \frac{1}{f^2}$$

$$S = k\frac{1}{f^2}$$

When f=8, S=125

$$125 = k \cdot \frac{1}{64}$$

$$k = 125 \times 64 = 8000$$

Therefore

$$S = 8000 \cdot \frac{1}{f^2} \quad \left(or\ S = \frac{8000}{k^2}\right)$$

(b) When f=4

$$S = 8000 \cdot \frac{1}{16}$$

$$= 500$$

Question 16

$$\frac{1}{2}(3.2)(8.4)\sin B = 10$$

$$\sin B = \frac{20}{(3.2)(8.4)}$$

Using the cosine rule

$$AC^2 = 3.2^2 + 8.4^2 - 2(3.2)(8.4)\sin B$$

$$AC^2 = 3.2^2 + 8.4^2 - 2 \times 20$$

$$AC^2 = 6.387...cm$$

So perimeter

$$= 3.2 + 8.4 + 6.387 = 17.98 = 18 \text{ to 3 sig figs}$$

Question 17

(a) Using Pythagoras

$$(x+8)^2 = x^2 + (x+5)^2$$

$$x^2 + 16x + 64 = x^2 + x^2 + 10x + 25$$

$$x^2 - 6x - 39 = 0$$

(b)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(-39)}}{2}$$

$$x = 9.928...., \quad -3.928....$$

So radius of circle, to 3 sig figs

$$= 9.93cm$$

Question 18

Need to construct $\triangle BDE$
BD is diagonal of ABCD
Using Pythagoras

$$BD^2 = 60^2 + 60^2 = 7200$$

$$BD = 84.8528.....cm$$

Length of AE is given by

$$\cos 30 = \frac{60}{AE}$$

$$AE = \frac{60}{\cos 30} = \frac{60 \times 2}{\sqrt{3}}$$

$$= \frac{120}{\sqrt{3}}$$

Using Pythagoras

$$DE^2 = 60^2 + \left(\frac{120}{\sqrt{3}}\right)^2$$

$$DE = 91.651.....$$

So required angle EDB is given by

$$\cos(EDB) = \frac{BD}{DE}$$

$$= 0.9258.....$$

$$EDB = 22.2^\circ \text{ to 1 d.p.}$$

Question 19

(a) Inserting $y = 6$ into

$$x^2 + y^2 = 25$$

$$x^2 + 36 = 25$$

$$x^2 = -11$$

Therefore no solns. of x $\Rightarrow y = 6$ does <u>not</u> cut the curve

(b) Inserting $y = 2x - 2$ into

$$x^2 + y^2 = 25$$

$$x^2 + (2x-2)^2 = 25$$

$$x^2 + 4x^2 - 8x + 4 = 25$$

$$5x^2 - 8x - 21 = 0$$

$$(5x+7)(x-3) = 0$$

First bracket gives

$$5x + 7 = 0 \Rightarrow x = -\frac{7}{5}$$

and

$$y = 2\left(-\frac{7}{5}\right) - 2 = -4.8$$

Second bracket gives

$$x - 3 = 0 \Rightarrow x = 3$$

and

$$y = 2(3) - 2 = 4$$

Question 20

(a) Upper bound in terms of m/s

$$\frac{400.005}{59.9}$$

Upper bound in terms of km/h

$$\frac{400.005}{59.9} \times \frac{3600}{1000}$$
$$= 24.040km/h$$

Lower bound in terms of m/s

$$\frac{399.995}{60.1}$$

Lower bound in terms of m/s

$$\frac{399.995}{60.1} \times \frac{3600}{1000}$$
$$= 23.960km/h$$

(b) Martin's speed

$$= \frac{1}{2}(23.960 + 24.040) = 24km/h$$

Midway between the uper bound and lower bound

(c)

Age Group	0-16	17-29	30-44	45-59	60+	Total
No. of people in sample	18	12	9	9	2	

Question 21

(a)

$$\frac{40 - x}{3} = 4 + x$$
$$40 - x = 12 + 3x$$
$$4x = 28$$
$$x = 7$$

(b)

$$\frac{4x^2 - 6x}{4x^2 - 9}$$
$$= \frac{2x(2x - 3)}{(2x + 3)(2x - 3)}$$
$$= \frac{2x}{2x + 3}$$

Question 22

Equating distance from x-axis to distance from $(0, 2)$

$$y = \sqrt{(y - 2)^2 + x^2}$$
$$y^2 = (y - 2)^2 + x^2$$
$$y^2 = y^2 - 4y + 4 + x^2$$
$$4y = x^2 + 4$$
$$y = \frac{1}{4}x^2 + 1$$

Paper 5 (Non-Calculator), 8 June 2004

Question 1

Cheese

$$\frac{2}{5} \times 70 = 28g$$

Topping

$$\frac{3}{5} \times 70 = 42g$$

Question 2

$$12\frac{1}{2} \div \frac{5}{8}$$
$$= \frac{25}{2} \div \frac{5}{8}$$
$$= \frac{25}{2} \times \frac{8}{5}$$
$$= \frac{5}{1} \times \frac{4}{1}$$
$$= \frac{20}{1} = 20$$

Question 3

(a)

$$p(q - p^2)$$
$$pq - p^3$$

(b)

$$5(3p + 2) - 2(5p - 3)$$
$$15p + 10 - 10p + 6$$
$$5p + 16$$

Question 4

(a) (i)

$$4 \times 10^7$$

(ii)

$$0.00003$$

(b) Using results from (a), expression becomes

$$3 \times 10^{-5} \times 4 \times 10^7$$
$$= 12 \times 10^2$$
$$= 1.2 \times 10^3$$

Question 5

(a)

$$\frac{1}{2}(180 - x)$$

(b)

$$3p + q = 11 \quad (5)$$
$$p + q = 3 \quad (6)$$

(5) - (6)

$$2p = 8$$
$$p = 4$$

Inserting $p = 4$ into (6)

$$4 + q = 3$$
$$q = -1$$

Question 6

The exterior angles sum to 360°

A single exterior angle of a hexagon will be

$$\frac{360}{6} = 60°$$

Question 7

There are several ways of doing this. From the way the line is presented, I would assume that this the method they are suggesting

Choose a point O somewhere 'inside' the angle. Set the compasses to OA and draw a circle with compass point on O, such that it cuts the line at B. Join B and O and extend to circumference of circle at C. Join A and C and this will be the perpendular to A (because angle at A will be the angle subtended by diameter BC). You can then bisect this right angle - with compass point on O, draw two arcs intersecting OC and OB. With compass points on these intersections draw two more arcs intersecting each other. Draw a line connecting this latter intersection and O.

Could also use the following method, although you might not have too much space -

Construct a perpendicular to the given line. Easiest to construct a perpendicular bisector. Set compasses to a width greater than half the length of the line. With the point of compasses on ends of line in turn, construct arcs above and below line so that you have two sets of intersecting arcs.

Connect these intersections to form the perpendicular bisector. Next, set compasses to length between A and point where bisector crosses the line, Use this distance to mark off a point C the same distance along the bisector (using compasses, with its point on aforementioned point where bisector crosses line). Connect C with A.

Question 8

Volume equals the area of the cross-section multiplied by length

$$\frac{1}{2} \times 3 \times 4 \times 7$$

$$= 42cm^2$$

Question 9

(a) (i)

$$\frac{x^6}{x^2} = x^4$$

(ii)

$$\left(y^4\right)^3 = y^{12}$$

(b)

$$(t+4)(t-2)$$

$$t^2 - 2t + 4t - 8$$

$$t^2 + 2t - 8$$

(c)

$$-2, -1, 0, 1, 2, 3$$

(d) (i)

$$36^{-\frac{1}{2}}$$

$$= \frac{1}{36^{\frac{1}{2}}}$$

$$= \frac{1}{6}$$

(ii)

$$27^{\frac{2}{3}}$$

$$= \left(27^{\frac{1}{3}}\right)^2$$

$$= 3^2$$

$$= 9$$

Question 10

(a) Draw the line $x = 2$: $x \geq 2$ will be the area to the right of this line

Draw the line $y = x$ (i.e. a straight line thru the origin at 45°): $y \geq x$ will be the area above this line.

Draw the line $y = -x + 6$ (i.e. crossing the y-axis at y=6 and with a negative gradient of 1: $y \leq -x+6$ (i.e. $x + y \leq 6$) will be the area below this line.

Shade in area that satisfies all three inequalities above simultaneously - i.e. the area bounded by a triangle with vertices (2,2), (2,4) and (3,3).

(b)

$$(2,4), (2,3), (2,2), (3,3)$$

inequality signs here indicate that points on lines are actually included within the region R

Question 11

$$D = ut + kt^2$$

$$ut = D - kt^2$$

$$u = \frac{D - kt^2}{t}$$

Question 12

(a)

$$25 < A \leq 35$$

(b)

$15 < A \leq 25$	44
$15 < A \leq 35$	100
$15 < A \leq 45$	134
$15 < A \leq 55$	153
$15 < A \leq 65$	160

(c) Draw cumulative frequency table, using information from (b)

(d) (i)

30

(ii)

$24 - 41$

(e) Draw a box stretching from 30 to 44. A vertical line within the box at 38 indicates the median. From both ends of the box project horizontal lines which terminates at a small vertical line at 24, to the left, and at a small vertical line at 54, to the right.

Question 13

(a)

$$\angle BCA = 55°$$

AC, being a diameter wii subtend a right angle at the circumference (at B), so $\angle BCA = 180 - 90 - 35$

(b) $\angle ABD = 55°$, so

$$\angle DBC = 90 - 55 = 35°$$

(c)

$$\angle BOA = 110°$$

this is twice $\angle BCA$ (from (a)) which is the angle AB subtends at the circumference - and so the angle it subtends at center will be twice this

Question 14

Using Pythagoras

$$x^2 + (2x)^2 = 25^2$$
$$x^2 + 4x^2 = 25^2$$
$$5x = \left(5^2\right)^2$$
$$5x = 5^4$$
$$x = 5^3 = 125\ cm^2$$

Question 15

(a) PS=QR because they are parallel and both have their end points on the same two parallel lines
PQ=SR from the same reasoning
the third side is shared (the diagonal of the parallelogram)

(b) $\angle SPQ$ is obtuse $\Rightarrow \angle SRQ$ is also obtuse

But for a cyclic quadrilateral, opposite angles need to sum to 180°

Question 16

Each vertical square corresponds to a unit of 0.8

(a)

Price	Frequency
$0 < P \leq 5$	40
$5 < P \leq 10$	60
$10 < P \leq 20$	56
$20 < P \leq 40$	32

(b)

Range	Height of Column
$0 < P \leq 5$	16
$5 < P \leq 10$	4
$10 < P \leq 20$	2.4
$20 < P \leq 40$	4.8

using the same convention as before where each (small) vertical square corresponds to 0.8, the heights of each column would be respectively

20 squares, 5 squares, 3 squares, 6 squares

Question 17

(a)

$$\frac{6}{\sqrt{2}}$$
$$= \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}}$$
$$= \frac{6\sqrt{2}}{2}$$
$$= 3\sqrt{2}$$

(b) Area of triangle

$$= \frac{1}{2} \times \frac{6}{\sqrt{2}} \times \frac{6}{\sqrt{2}}$$
$$= \frac{36}{4}$$
$$= 9\ cm^2$$

Question 18

Type of Proportionality	Graph Letter
y is directly proportional to x	D
y is inversely proportional to x	A
y is proportional to the square of x	B
y is inversely proportional to the square of x	C

Question 19

(a)

$$5n$$

(b) (i)

$$5n + 5(n+1)$$
$$= 5n + 5n + 5$$
$$= 10n + 5$$

10 is always even, and so $10n + 5$ must always be odd

(ii)

$$5n \times (5(n+1))$$
$$= 5n(5n+5)$$
$$= 25n^2 + 25n$$
$$= 25n(n+1)$$

Since either n or $(n+1)$ will be even, the whole expression will be even

Question 20

(a)

$$a = 2, \;\; b = -1$$

(b)

$$0°,\ 360°,\ 720°$$

(c) This will be the same as when $x = 90°$, i.e.

$$3$$

Question 21

(i)

$$\vec{PQ} = \mathbf{q} - \mathbf{p}$$

$$\vec{OS} = \vec{OP} + \frac{1}{2}\vec{PQ}$$

$$\vec{OS} = \mathbf{p} + \frac{1}{2}(\mathbf{q} - \mathbf{p})$$

$$\vec{OS} = \mathbf{p} + \frac{1}{2}\mathbf{q} - \frac{1}{2}\mathbf{p}$$

$$\vec{OS} = \frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$$

$$\vec{OS} = \frac{1}{2}(\mathbf{p} + \mathbf{q})$$

(ii)

$$\vec{RS} = \vec{OS} - \vec{OR}$$

$$\vec{RS} = \frac{1}{2}(\mathbf{p} + \mathbf{q}) - \frac{1}{2}\mathbf{p}$$

$$\vec{RS} = \frac{1}{2}\mathbf{q}$$

thus RS is parallel to OQ

Question 22

$$\frac{2}{x+1} + \frac{3}{x-1} = \frac{5}{x^2-1}$$

$$2(x-1) + 3(x+1) = 5$$

$$2x - 2 + 3x + 3 = 5$$

$$5x = 4$$

$$x = \frac{4}{5}$$

Using $(x^2 - 1) = (x + 1)(x - 1)$

Question 23

(a) (i) Area of sector

$$= \frac{\pi x^2}{3}$$

(ii) Arc length of this sector

$$= \frac{2\pi x}{3}$$

(b)

$$A = \frac{\pi x^2}{3} \;\text{ from (a)}$$

also

$$A = \pi r x \Rightarrow r = \frac{A}{\pi x} \Rightarrow r = \frac{\frac{\pi x^2}{3}}{\pi x} \Rightarrow r = \frac{x}{3}$$

now

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \left(\frac{x}{3}\right)^2 h$$

and since V=3A, from the question

$$\frac{1}{3}\pi \left(\frac{x^2}{9}\right) h = 3\left(\frac{\pi x^2}{3}\right)$$

$$\frac{1}{27}h = 1$$

$$h = 27\ cm$$

Paper 6 (Calculator), 9 November 2004

Question 1

$$40\% \; of \; 240 = 96$$

$$70\% \; of \; 200 = 140$$

so percentage of all students who went to party

$$\frac{236}{440} \times 100 = 53.\dot{6}\dot{3}$$

Question 2

£1000 corresponds to $2100cm^2$ If length is $48cm$, then width

$$= \frac{2100}{48}$$
$$= 43.75 \; cm$$

Question 3

$\mathbf{x}$	$\mathbf{x^3 + 4x}$
4	80
5	145
4.5	109.125
4.3	96.707
4.4	102.784
4.35	99.512875

so

$$x = 4.4 \text{ to 1 dec place}$$

must *calculate expression for 4.35 to see whether the answer is 4.3 or 4.4 to one decimal place*

Question 4

(a)

$$4(2x + 1) = 2(3 - x)$$
$$8x + 4 = 6 - 2x$$
$$10x = 2$$
$$x = \frac{2}{10} = \frac{1}{5}$$

(b)

$$2p^2 - 4pq$$
$$2p(p - 2q)$$

(c)

$$x^2 + 7x + 6$$
$$(x + 1)(x + 6)$$

Question 5

(a) Value

$$= 8000\,(1.05)^3$$
$$= £9261$$

(b)

$$A$$

(c)

$$x(1.05) = 3885$$
$$x = \frac{3885}{1.05}$$
$$= £3700$$

You could always work out the compound interest the 'long' way, but I would recommend mastering this method

Question 6

(a) Average Speed

$$= \frac{200}{21.2}$$
$$= 9.433962264150943396226415094339 6 \; m/s$$

number of figures depends on your calculator

(b)

$$9.43 \; m/s$$

the input data has three significant figures, so answer should have a maximum of three significant figures

Question 7

(a)

$$\frac{30}{100} = 0.3$$

(b)

$$250 \times 0.7 = 175$$

Question 8

$$\tan x = \frac{5}{12.5} = 0.4$$
$$x = 21.8014....$$
$$= 21.8cm \text{ to 1 dec. place}$$

Question 9

Using Pythagoras

$$17^2 = 10^2 + CD^2$$

$$CD^2 = 17^2 - 10^2$$

$$= 189$$

$$CD = 13.7477.....$$

$$CD = 13.7 \text{ to 1 dec. pl.}$$

Question 10

(a)

$$y = \sqrt{\frac{r + t\sin x^\circ}{r - t\sin x^\circ}}$$

$$= \sqrt{\frac{8.8 + 7.2\sin 40^\circ}{8.8 - 7.2\sin 40^\circ}}$$

$$= 1.794065....$$

$$= 1.79 \text{ to 3 sig figs}$$

(b)

$$2 = \sqrt{\frac{r+5}{r-5}}$$

$$4 = \frac{r+5}{r-5}$$

$$4(r-5) = r + 5$$

$$4r - 20 = r + 5$$

$$3r = 25$$

$$r = \frac{25}{3}$$

Question 11

$$\mathbf{L_1} : \text{y} = 2\text{x} + 3$$

This has a gradient of 2
so gradient of $\mathbf{L_2}$ is also 2, giving

$$y = 2x + c$$

At (3,2)

$$2 = 2(3) + c$$

$$2 = 6 + c$$

$$c = 2 - 6 = -4$$

so equation of $\mathbf{L_2}$ is

$$y = 2x - 4$$

Question 12

(a) No of videos watched by all 60 members

$$= 2.8 \times 60 = 168$$

No of videos watched by all boys

$$= 3.3 \times 40 = 132$$

No of videos watched by girls

$$= 168 - 132 = 36$$

So mean number watched by girls

$$= \frac{36}{20} = 1.8$$

(b) The lists must be of the same size

p and q are equal

Question 13

(a) Equating corresponding sides of similar triangles

$$\frac{CD}{10} = \frac{5}{4}$$

$$CD = \frac{50}{4}$$

$$= 12.5cm$$

(b)

$$\frac{ED + 4.8}{10} = \frac{4.8}{4}$$

$$ED + 4.8 = \frac{48}{4}$$

$$ED = 12 - 4.8$$

$$ED = 7.2cm$$

Question 14

(a)

$$x^2 + x + 11 = 14$$

completing square

$$\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + 11 = 14$$

$$\left(x + \frac{1}{2}\right)^2 = 3.25$$

$$x + \frac{1}{2} = \pm\sqrt{3.25}$$

$$x = -\frac{1}{2} \pm \sqrt{3.25}$$

so

$$x = 1.3027... = 1.30 \text{ to 3 sig figs}$$

or

$$x = -2.3027... = -2.30 \text{ to 3 sig figs}$$

(b) When x=10, y =121 which is not prime

Question 15

(a) Using Pythagoras on $\triangle$ ABC

$$AC^2 = 5^2 + 7^2$$

$$= 25 + 49 = 74$$

$$AC = \sqrt{74}$$

Using Pythagoras on $\triangle$ ACG

$$AG^2 = (\sqrt{74})^2 + 3^2$$

$$= 74 + 9 = 83$$

$$AG = 9.1104... = 9.11 \text{ to 3 sig figs}$$

(b) *Need to calculate angle between AG and AC*

$$\sin(\angle GAC) = \frac{3}{9.11}$$

$$\angle GAC = 19.226.... \quad = 19.2 \text{ to 1 dec pl.}$$

Question 16

$$T \ \alpha \ R^2$$

$$T = kR^2$$

Inserting given data

$$32 = k(120)^2$$

$$k = \frac{32}{120^2}$$

$$k = \frac{2 \times 16}{(4 \times 30)^2}$$

$$k = \frac{2}{(30)^2} = \frac{1}{450}$$

could always use a calculator if you must

When R = 150

$$T = \frac{1}{450}(150)^2$$

$$T = 50 \ mins$$

Question 17

Scale factor

$$= \sqrt{\frac{800}{450}} = \frac{\sqrt{800}}{\sqrt{450}} = \frac{\sqrt{10 \times 5 \times 16}}{\sqrt{10 \times 5 \times 9}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

so volume of Y

$$= 1350 \times \left(\frac{4}{3}\right)^3$$

$$= 3200 \ cm^3$$

Question 18

For Upper Bound of T

$$\sqrt{\frac{L}{g}} = \sqrt{\frac{1.365}{9.75}}$$

For Lower Bound of T

$$\sqrt{\frac{L}{g}} = \sqrt{\frac{1.355}{9.85}}$$

Subtracting expression for Lower Bound from expression from Upper Bound, and multiplying by 6.283

$$0.02054.... \quad = 0.021 \text{ to 2 sig figs}$$

Question 19

$$\frac{4x^2 - 9}{2x^2 - 5x + 3}$$

$$\frac{(2x + 3)(2x - 3)}{(2x - 3)(x - 1)}$$

$$\frac{2x + 3}{x - 1}$$

Question 20

(a) Need

$$P((Gary \ Wins \ and \ Gary \ Doesn't \ Win) \ OR$$

$$(Gary \ Doesn't \ Win \ and \ Gary \ Wins))$$

$$= (0.55 \times 0.45) + (0.45 \times 0.55)$$

$$= 2(0.55 \times 0.45)$$

$$= 0.495$$

(b) Probability of Gary losing = 0.15

There are only three scenarios to be considered - the symmetrical situation where they both win one and lose one, or when both games are drawn

Need

$$P((Gary \ Wins \ and \ Gary \ Loses) \ OR$$

$$(Gary \ Loses \ and \ Gary \ Wins) \ OR \ (Both \ Draws))$$

$$= (0.55 \times 0.15) + (0.15 \times 0.55) + (0.3 \times 0.3)$$

$$= 0.255$$

Question 21

(a)

$$x^2 + y^2 = 100 \quad i.e. \quad x^2 + y^2 = 10^2$$

is a circle based on the origin with a radius of 10

$$2y = 3x - 4$$

is a straight line with a gradient of 1.5 and intersects the y-axis at -2

(b)

$$(6.4, 7.7), \quad (-4.6, -8.9)$$

or similar

(c) Completing the square

$$x^2 + 6x = (x+3)^2 - 9 \Rightarrow q = 9$$

(d) Completing the square

$$y^2 - 4y = (y-2)^2 - 4$$

so

$$x^2 + 6x + y^2 - 4y - 87 = 0$$

$$\Rightarrow (x+3)^2 - 9 + (y-2)^2 - 4 - 87 = 0$$

$$\Rightarrow (x+3)^2 + (y-2)^2 = 100$$

so $x = 3$ and $y = 10$ satisfy this equation

$(x+3) \Rightarrow$ translate $x = 6$ by 3 units in a negative direction, and $(y-2) \Rightarrow$ translating $y = 8$ by 2 units in a positive direction

Question 22

(a) Area of triangle

$$= \frac{1}{2} \times 10 \times 12 \times \sin 45^\circ$$

$$= 60 \times \sin 45^\circ$$

$$= 42.4264....$$

$$= 42.4 \ m^2 \text{ to 3 sig figs}$$

(b) Area of $\triangle ABC$

$$\frac{1}{2} \times BC \times h$$

Area of $\triangle ACD$

$$\frac{1}{2} \times CD \times h$$

so

$$\frac{Area\ of\ triangle\ ABC}{Area\ of\ triangle\ ACD} = \frac{\frac{1}{2} \times BC \times h}{\frac{1}{2} \times CD \times h} = \frac{BC}{CD}$$

(c) If height of XYZ is h

Area of $\triangle YXW$

$$= \frac{1}{2} \times XY \times XW \times \sin \angle YXW = \frac{1}{2} YW \times h$$

$$\Rightarrow \sin \angle YXW = \frac{\frac{1}{2} YW \times h}{\frac{1}{2} \times XY \times XW} = \frac{YW \times h}{XY \times XW}$$

Area of $\triangle WXZ$

$$= \frac{1}{2} \times XZ \times XW \times \sin \angle WXZ = \frac{1}{2} WZ \times h$$

$$\Rightarrow \sin \angle WXZ = \frac{\frac{1}{2} WZ \times h}{\frac{1}{2} \times XZ \times XW} = \frac{WZ \times h}{XZ \times XW}$$

Since $\sin \angle YXW = \sin \angle WXZ$

$$\frac{YW \times h}{XY \times XW} = \frac{WZ \times h}{XZ \times XW}$$

$$\frac{YW}{XY} = \frac{WZ}{XZ}$$

$$\frac{YW}{WZ} = \frac{XY}{XZ}$$