Edexcel **GCSE Mathematics (1387)**

Higher Tier 2003

Model Answers

In general, the number of significant figures in an answer should not exceed the number of significant figures in the input data, or if this data has differing numbers of significant figures, the data with the lowest number of significant figures.

Brian Daugherty

Statements in italics are for information rather than a part of the answer

Paper 5 (Non-Calculator), 4 June 2003

Question 1

(i)

$$119.31$$

(ii)

$$119\ 310$$

(iii)

$$1.23$$

Question 2

$$10\%\ of\ 12000 = 1200$$

Therefore value of car after 1 year

$$= 12000 - 1200 = 10800$$

Now

$$10\%\ of\ 10800 = 1080$$

Therefore value of car after 2 years

$$= 10800 - 1080 = £9720$$

Question 3

$$7r + 2 = 5(r - 4)$$

$$7r + 2 = 5r - 20$$

$$2r = -22$$

$$r = -11$$

Question 4

(a)

$$-1,\ 0,\ 1$$

(b) Insert lines corresponding to
$x = -2$
$x = 1$
$y = -2$
$y = x + 1$

giving the required points, as follows

$$(1, 1), (1, 0), (1, -1), (0, 0), (0, -1), (-1, -1)$$

Question 5

Constant first difference of 5 $\Rightarrow$ the required expression is of the form

$$5n + k$$

where k is a constant

On inspection $k = 1$. So expression is

$$5n + 1$$

Question 6

(a) D has vertices

$$(0, 0), (3, 0), (0, -2)$$

(b) rotation by 180° (anti-clockwise or clockwise) about (0,1)

Question 7

Draw bisector of BC through A

Draw an arc of 5cm from A (using compasses)

Shaded area will be 'above' bisector and to the 'left' of arc.

Question 8

$$x + y + z \ : \ Length$$

$$xyz \ : \ Volume$$

$$xy + yz + xz \ : \ Area$$

Question 9

(a) A 'short' question could be on the lines : list, in order, your three favourite genres of food you like to eat in a restaurant? (any attempt to include all possibilities and not 'lead' the respondents on will be satisfactory.

(b)
1. His family is not representative of the population as a whole
2. The question is directing the respondents to a particular answer

Question 10

(a) Distance travelled

$$= 8 \times 10^4 \times 6 \times 10^2$$
$$= 48 \times 10^6$$
$$= 4.8 \times 10^7 km$$

(b)

$$(2 \times 10^5) + (3 \times 10^4)$$
$$= (20 \times 10^4) + (3 \times 10^4)$$
$$= 23 \times 10^4$$
$$= 230\ 000 km$$

Question 11

(a)

$$(x+y)^2$$
$$= (x+y)(x+y)$$
$$= x^2 + 2xy + y^2$$

(b) Expression can be stated as

$$(3.47 + 1.53)^2$$
$$= 5^2 = 25$$

Question 12

(i) Angle ACB = 27°, because FE is a tangent to the circle, and therefore angle ACE = 90°.

(ii) Angle BAC = 63°, because (1) the diameter AC subtends an angle of 90° at the circumference and (2) from (i) we know that angle ACB is 27°.

Question 13

(i)

$$(p^3)^3 = p^9$$

(ii)

$$\frac{3q^4 \times 2q^5}{q^3} = \frac{6q^9}{q^3} = 6q^6$$

Question 14

(a) (i) lower quartile = 152

(ii) upper quartile = 177

(b) Box plot consisting of left hand side of box at 152 and right-hand side at 177. A line representing the median is drawn at 167. Lines are extended to display the range - to the left to 132 and to the right to 182.

Question 15

$$40° = \frac{40}{360} \times 2\pi\ radians$$

Length of arc

$$= 9 \times \frac{40}{360} \times 2\pi = 2\pi$$

So total perimeter

$$= (18 + 2\pi)\, cm$$

Question 16

(i)

$$4^0 = 1$$

(ii)

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

(iii)

$$16^{\frac{3}{2}} = \left(16^{\frac{1}{2}}\right)^3 = 4^3 = 64$$

Question 17

(a)

$$F\ \alpha\ \frac{1}{x^2}$$
$$F = \frac{k}{x^2}$$

When x=3, F=4

$$4 = \frac{k}{9} \Rightarrow k = 36$$

so

$$F = \frac{36}{x^2}$$

(b)

$$F = \frac{36}{4} = 9$$

(c)

$$64 = \frac{36}{x^2} \Rightarrow x^2 = \frac{36}{64} \Rightarrow x = \frac{6}{8} = \frac{3}{4}$$

Question 18

$$\frac{\left(5+\sqrt{3}\right)\left(5-\sqrt{3}\right)}{\sqrt{22}}$$
$$= \frac{25-3}{\sqrt{22}}$$
$$= \frac{22}{\sqrt{22}}$$
$$= \sqrt{22}$$

Question 19

(a)
$10 \le x \le 25 \quad 60$
$25 \le x \le 30 \quad 40$

(b) Between 30 and 40, 2.5 units high

Between 40 and 70, 10 units high

Question 20

(a)

$$2(3x+4)-3(4x-5)$$
$$=6x+8-12x+15$$
$$=-6x+23$$

(b)

$$\left(2xy^3\right)^5$$
$$=2^5x^5y^{15}$$
$$=32x^5y^{15}$$

(c)

$$\frac{n^2-1}{n+1}\times\frac{2}{n-2}$$
$$=\frac{(n-1)(n+1)}{n+1}\times\frac{2}{n-2}$$
$$=\frac{(n-1)}{1}\times\frac{2}{n-2}$$
$$=\frac{2(n-1)}{(n-2)}$$

Question 21

Multiply all coordinates by 1.5, so that apexes transform so

$$(2,1)\to(-3,-1.5)$$
$$(2,3)\to(-3,-4.5)$$
$$(3,1)\to(-4.5,-1.5)$$

Question 22

P(2 black)

$$=P(B,B,NB)+P(B,NB,B)+P(NB,B,B)$$
$$=3\left(\frac{3}{10}\times\frac{3}{10}\times\frac{7}{10}\right)$$
$$=3\times\frac{63}{1000}=\frac{189}{1000}$$

P(2 red)

$$=P(R,R,NR)+P(R,NR,R)+P(NR,R,R)$$
$$=3\left(\frac{5}{10}\times\frac{5}{10}\times\frac{5}{10}\right)$$
$$=3\times\frac{125}{1000}=\frac{375}{1000}$$

P(2 green)

$$=P(G,G,NG)+P(G,NG,G)+P(NG,G,G)$$
$$=3\left(\frac{2}{10}\times\frac{2}{10}\times\frac{8}{10}\right)$$
$$=3\times\frac{32}{1000}=\frac{96}{1000}$$

So total probability

$$=\frac{189}{1000}+\frac{375}{1000}+\frac{96}{1000}$$
$$=\frac{660}{1000}$$

Question 23

(a) (i)

$$\vec{AB}=6\mathbf{b}-6\mathbf{a}=6(\mathbf{b}-\mathbf{a})$$

(ii)

$$\vec{EF}=6\mathbf{a}$$

(b)

$$\vec{EB}=2\times6\mathbf{b}=12\mathbf{b}$$
$$\vec{BC}=-6\mathbf{a}$$

so

$$\vec{EX}=12\mathbf{b}+\frac{1}{2}(-6\mathbf{a})=12\mathbf{b}-3\mathbf{a}$$

(c)

$$\vec{BY}=\frac{2}{3}\times6(\mathbf{b}-\mathbf{a})=4(\mathbf{b}-\mathbf{a})$$
$$\vec{EY}=12\mathbf{b}+4(\mathbf{b}-\mathbf{a})$$
$$=12\mathbf{b}+4\mathbf{b}-4\mathbf{a}$$
$$=16\mathbf{b}-4\mathbf{a}$$
$$=\frac{4}{3}(12\mathbf{b}-3\mathbf{a})$$

so $\vec{EX}$ and $\vec{EY}$ lie along the same line

Question 24

(a) (i)

$$(5, -4)$$

(ii)

$$(2, -9)$$

(iii)

$$(2, 4)$$

(iv)

$$(2, -4)$$

(b) The general form of equation is

$$y = ax^2 + bx + c$$

At (0,0)

$$0 = a(0)^2 + b(0) + c \Rightarrow c = 0$$

At (2,-4)

$$-4 = a(2)^2 + b(2) \Rightarrow 4a + 2b = -4 \qquad (1)$$

By symmetry, also crosses x-axis at (4,0), giving

$$0 = a(4)^2 + b(4) \Rightarrow 16a + 4b = 0 \qquad (2)$$

4×(1) - (2)

$$4b = -16 \Rightarrow b = -4$$

and

$$4a + 2(-4) = -4$$

$$4a = 4$$

$$a = 1$$

so

$$f(x) = x^2 - 4x$$

Paper 6 (Calculator), 10 June 2003

Question 1

(a) Volume of cylinder

$$= \pi r^2 h$$
$$= \pi(4^2)(10)$$
$$= 160\pi = 503 cm^2$$

(b) Consider a right-angled triangle of sides 10 cm and 8 cm

$$\text{Hypotenuse}^2 = 10^2 + 8^2$$
$$= 100 + 64 = 164$$
$$\text{Hypotenuse} = 12.806...cm$$

So a pencil of 13cm cannot fit inside the cylinder

Question 2

(a) (i)

$$60 = 2 \times 30$$
$$= 2^2 \times 15$$
$$= 2^2 \times 3 \times 5$$

(ii)

$$96 = 2 \times 48$$
$$= 2^2 \times 24$$
$$= 2^3 \times 12$$
$$= 2^4 \times 6$$
$$= 2^5 \times 3$$

(b) HCF

$$= 2^2 \times 3 = 12$$

(c) LCM

$$= 2^5 \times 3 \times 5 = 480$$

Question 3

(a) Median = hald the 20th ans 21st items, which will fall in the class $150 < C \leq 200$

(b) One extra item at *£1000* will raise the median to be the 21st item, which lies in the same class as before

(c)

$$5200 \times \frac{100}{80} = £6500$$

Question 4

(a) Volume of cuboid

$$= x \times x \times (x+1) = x^3 + x^2$$

Therefore

$$x^3 + x^2 = 230$$

(b)

5	150	6	252
5.5	196.625		
5.7	217.683		
5.8	228.752		
5.9	240.189		
5.85	234.424125		

So answer to 1 d.p. is 5.9

Question 5

Area

$$= \frac{\pi r^2}{2} = \frac{\pi 7.5^2}{2} = 88.4 cm^2$$

Question 6

(a)

$$5 = \frac{1}{2}x + 1$$
$$\frac{1}{2}x = 4$$
$$x = 8$$

(b)

$$y = \frac{1}{2}x + 3$$

for example (any constant on the end will suffice)

(c)

$$y = \frac{1}{2}x + 1$$
$$\frac{1}{2}x = y - 1$$
$$x = 2(y-1)$$

Question 7

$$2x - 3y = 11 \quad (3)$$

$$5x + 2y = 18 \quad (4)$$

(3) $\times 2$

$$4x - 6y = 22 \quad (5)$$

(4) $\times 3$

$$15x + 6y = 54 \quad (6)$$

Add (5) and (6)

$$19x = 76$$

$$x = 4$$

therefore

$$2(4) - 3y = 11$$

$$3y = -3$$

$$y = -1$$

Question 8

(a)

$$\frac{CD}{4.8} = \frac{10}{6}$$

$$CD = \frac{48}{6} = 8cm$$

(b)

$$fracCA4.5 = \frac{10}{6}$$

$$CA = \frac{45}{6} = 7.5$$

Perimeter

$$= 8 + 4.8 + 4 + 3 = 19.8cm$$

Question 9

$$y^2 = \frac{(3 \times 10^8)(2 \times 10^7)}{(3 \times 10^8) + (2 \times 10^7)}$$

$$= \frac{6 \times 10^{15}}{32 \times 10^7}$$

$$= 0.1875 \times 10^{15}$$

$$= 1.875 \times 10^{14}$$

$$y = 1.4 \times 10^7$$

Question 10

$$\tan 38 = \frac{AB}{8.5}$$

$$AB = 8.5 \times \tan 38$$

$$AB = 6.64cm$$

Question 11

(a) For a fair dice, we would expect 100 sixes. 200 sixes is too divergent a result, so dice is not fair.

(b) Left-hand side of tree requires a probability of 5/6 o n the bottom branch. The right-hand side will have two branches from each node. Each set of branches will 'mirror' those on the left-hand side with a probability of 1/6 for a six and 5/6 for a 'not six'.

(c) (i)

$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

(ii) P(no six)

$$= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

so P(at least one six)

$$= 1 - \frac{25}{36} = \frac{11}{36}$$

Question 12

(a) Volume of large cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(7.5)^2(30)$$

$$= 562.5\pi$$

Volume of cone A

$$= \frac{1}{3}\pi(2.5)^2(10)$$

$$= 20.8\dot{3}\pi$$

Volume of B

$$= 2250\pi - \frac{250}{3}\pi$$

$$= 541.\dot{6}\pi = 1701.696... = 1700 \text{ to 3 sig figs}$$

(Note: Although I have laid out intermediate figures here, I would in reality do it 'all in one go', storing figures in my calculator right through the calculation. This comment applies to other questions on this paper, as well)

(b)

$$S = 2\pi d\sqrt{h^2 + d^2}$$

$$S^2 = 4\pi^2 d^2(h^2 + d^2)$$

$$S^2 = 4\pi^2 d^2 h^2 + 4\pi^2 d^4$$

$$4\pi^2 d^2 h^2 = S^2 - 4\pi^2 d^4$$

$$h^2 = \frac{S^2}{4\pi^2 d^2} - d^2$$

$$h = \sqrt{\frac{S^2}{4\pi^2 d^2} - d^2}$$

(c) If d and l are lengths related to the smaller frustrum, the surface area of the larger frustrum is given by

$$S = 2\pi\left(\frac{3d}{2}\right)\sqrt{\left(\frac{3h}{2}\right)^2 + \left(\frac{3d}{2}\right)^2}$$

$$= 2\pi\left(\frac{3d}{2}\right)\sqrt{\frac{9}{4}(h^2+d^2)}$$

$$= 2\pi\left(\frac{3d}{2}\right)\left(\frac{3}{2}\right)\sqrt{(h^2+d^2)}$$

$$= \frac{9}{4}\left(2\pi d\sqrt{(h^2+d^2)}\right)$$

Therefore Surface Area of larger frustrum

$$= \frac{9}{4} \times 450 = 1012.5cm^2$$

Question 13

(a) Area of trapezium

$$\frac{x+20}{2} \times 2x = 400$$

$$x^2 + 20x = 400$$

(b) Rearrange

$$x^2 + 20x - 400 = 0$$

Using formula

$$x = \frac{-20 \pm \sqrt{20^2 - 4 \times 1 \times (-400)}}{2}$$

$$x = \frac{-20 \pm \sqrt{2000}}{2}$$

$$x = 12.361 \;\; and \;\; x = -32.361$$

So

$$x = 12.361cm$$

Question 14

(a) Area of ABC

$$= \frac{1}{2}(8)(15)\sin 70$$

$$= 56.4cm^2$$

(b) side AB is given by

$$AB^2 = 8^2 + 15^2 - 2(8)(15)\cos 70$$

$$AB = 14.384...$$

angle B is given by

$$\frac{\sin 70}{AB} = \frac{\sin B}{8}$$

$$\sin B = 0.5226...$$

Now

$$\sin B = \frac{CX}{15}$$

$$CX = \sin B \times 15 = 7.84cm$$

Question 15

(a)

$$(2a-1)^2-(2b-1)^2 = 4a^2-2a-2a+1-(4b^2-2b-2b+1)$$

$$= 4a^2 - 4a + 1 - 4b^2 + 4b - 1$$

$$= 4a^2 - 4a - 4b^2 + 4b$$

$$= 4(a^2 - b^2) - 4(a - b)$$

$$= 4(a-b)(a+b) - 4(a-b)$$

$$= 4(a-b)(a+b-1))$$

(b) If a and b are both odd or both even, then (a-b) will be a multiple of 2 and therefore, because of the coefficient of 4, the whole expression will be a multiple of 8.

If one of a,b is odd and the other even, then (a+b-1) will be a multiple of 2, and therefore employing similar logic to above, the whole expression will be a multiple of 8.

Question 16

(a) Upper bound of g is given by

$$\frac{2(4.505)}{(1.25^2)(\sin 29.5)}$$

$$= 11.710$$

Lower bound of g is given by

$$\frac{2(4.495)}{(1.35^2)(\sin 30.5)}$$

$$= 9.719$$

(b)

$$g = 10.714..... = 11 \;\; \text{to 2 significant figures}$$

given to 2 significant figures, because that is the lowest number of significant figures in the inputed data

Question 17

(a) (i)

$$2^{p+q} = 2^p 2^q = xy$$

(ii)

$$2^{2q} = 2^q 2^q = y^2$$

or

$$2^{2q} = (2^q)^2 = y^2$$

(iii)

$$2^{p-1} = \frac{2^p}{2} = \frac{x}{2}$$

(b)

$$xy = 32 \Rightarrow x = \frac{32}{y}$$

Inserting into second equation

$$2\left(\frac{32}{y}\right)y^2 = 32$$

$$64y = 32 \Rightarrow y = \frac{1}{2}$$

and

$$x\left(\frac{1}{2}\right) = 32 \Rightarrow x = 64$$

If $y = \frac{1}{2}$

$$2^q = \frac{1}{2} \Rightarrow q = -1$$

If $x = 64$

$$2^p = 64 \Rightarrow p = 6$$

Question 18

(a)

$$x^2 - 2mx = (x-m)^2 - k$$

$$k = x^2 - 2mx + m^2 - (x^2 - 2mx)$$

$$k = x^2 - 2mx + m^2 - x^2 + 2mx$$

$$k = m^2$$

(b) (i) Minimum value occurs when

$$(x-m)^2 = 0$$

giving min. value

$$= -k = -m^2$$

(ii) Minimum value occurs when

$$x = m$$

Question 19

If the events were independent then the probability that both Betty and Colin will be late

$$= 0.05 \times 0.06 = 0.003$$

Since the actual probability is 0.011, the events are **not** independent

Question 20

(i)

$$a = 50$$

(ii)

$$b = 50$$

(iii)

$$k = \frac{360}{\lambda} = \frac{360}{90} = 4$$