Cambridge **IGCSE Additional Mathematics**

2004

Model Answers

Note the instructions ask you to give answers to 3 sig figs, where appropriate. (In general, the number of significant figures in an answer should not exceed the number of significant figures in the input data, or if this data has differing numbers of significant figures, the data with the lowest number of significant figures).

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Statements in italics are for information rather than a part of the answer

Paper 1 - 0606/01, May/June 2004

Question 1

(i)

$$y = \frac{3x-2}{x^2+5}$$

$$\frac{dy}{dx} = \frac{(x^2+5)(3)-(3x-2)(2x)}{(x^2+5)^2}$$

$$= \frac{3x^2+15-(6x^2-4x)}{(x^2+5)^2}$$

$$= \frac{3x^2+15-6x^2+4x}{(x^2+5)^2}$$

$$= \frac{-3x^2+4x+15}{(x^2+5)^2}$$

(ii) When

$$\frac{dy}{dx} = 0$$

$$-3x^2+4x+15 = 0$$

$$3x^2-4x-15 = 0$$

$$(3x+5)(x-3) = 0$$

so

$$3x+5 = 0 \Rightarrow x = -\frac{5}{3}$$

$$x-3 = 0 \Rightarrow x = 3$$

Question 2

Equating the two expressions

$$x^3 = 5x-2$$

$$x^3-5x+2 = 0$$

By trial and error, inserting values into this expression $\underline{x=2}$ is a solution

$$\Rightarrow (x-2) \text{ is a factor}$$

Dividing the cubic by $x-2$ produces x^2+2x-1
Therefore an initial factorization of the cubic is

$$(x-2)(x^2+2x-1)$$

Using the formula on the quadratic factor

$$x = \frac{-2 \pm \sqrt{2^2-4(1)(-1)}}{2}$$

$$= -1 \pm \frac{\sqrt{8}}{2}$$

$$= -1 \pm \frac{2\sqrt{2}}{2}$$

$$x = -1 \pm \sqrt{2}$$

Question 3

(i) Draw graphs

Draw the straight line $y = 2x+3$ in first quadrant and into the second quadrant, as far as the x-axis at $x = -1.5$. The effect of taking the modulus is to 'mirror' this line in the second quadrant as you continue plotting to the left of $x = -1.5$.

$y = 1-x$ is a 'straightforward' straight line, crossing the y-axis at y=1, and with a gradient of -1, i.e. at $45°$ from bottom right to top left

(ii) Find the intersection of these two lines. Values are approx. $x = -0.9$ and $x = -3.8$

If you were to solve the problem algebraically, you would proceed as follows

$$|2x+3| = 1-x$$

$$(2x+3)^2 = (1-x)^2$$

and so on, producing values

$$x = -0.88036.. \text{ and } -3.786...$$

Question 4

(i)

$$a = 2$$

(the amplitude)

$$b = 3$$

stretch factor of $\frac{1}{3} \Rightarrow$ *b =3*

(ii)

$$c = 1$$

without c, the minimum value would be determined solely by a (the amplitude) - i.e. it would have a value of -2

(iii) Sketch graph. shape identical to a sine function with zeroes at 0, 120, 240 etc, except that it is translated one unit upwards, having maxima at 3 and minima at -1

Question 5

$$5y+2x = 1$$

$$5y = 1-2x$$

$$y = \frac{1-2x}{5}$$

Into $xy+24 = 0$

$$x\left(\frac{1-2x}{5}\right)+24 = 0$$

$$x(1-2x)+120 = 0$$

$$x-2x^2+120 = 0$$

$$2x^2 - x - 120 = 0$$
$$(2x+15)(x-8)$$

So
$$2x + 15 = 0 \Rightarrow x = \frac{-15}{2} = -7.5$$
$$y = \frac{1 - 2(-7.5)}{5} = \frac{16}{5}$$

and
$$x - 8 = 0 \Rightarrow x = 8$$
$$y = \frac{1 - 2(8)}{5} = \frac{-15}{5} = -3$$

So A and B have coordinates
$$(-7.5, 3.2), \quad (8, -3)$$

so
$$AB^2 = (15.5)^2 + (6.2)^2$$

and
$$AB = 16.69401....$$
$$= 16.7 \text{ to 1 dp}$$

Question 6

$$\begin{bmatrix} 4 & 6 & 8 \end{bmatrix} \begin{bmatrix} 0.6 & 0.5 \\ 0.3 & 0.4 \\ 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 300 \\ 240 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 5.2 \end{bmatrix} \begin{bmatrix} 300 \\ 240 \end{bmatrix}$$

$$= \$2448$$

Question 7

Since
$$\mathbf{v}_{\text{wind(apparent)}} = \mathbf{v}_{\text{wind(true)}} - \mathbf{v}_{\text{cycle}}$$

Construct a vector addition diagram, with a vector pointing due South representing the cyclist's velocity. To the head of this vector place another vector in a South West direction representing the 'apparent' wind. From the back of the first vector to the tip of the second vector, draw another vector representing the real wind (which has magnitude 12). Insert the 45° degree angle implied for the second vector, and therefore also the 135° angle inside the triangle formed by the vectors. Call the lower angle of this triangle x and designate the upper angle (the angle betwen the cyclist's velocity vector and the 'real wind' vector) as y.

Using Sine Rule
$$\frac{\sin x}{7} = \frac{\sin 135}{12}$$
$$\sin x = \frac{7 \sin 135}{12}$$
$$= (0.41247....)$$
$$x = 24.36065....$$

So the angle y at the top of the 'triangle
$$= 180 - (135 + 24.36065...)$$
$$= 20.6393..$$

By opposite angles, or otherwise, this is also the bearing of the direction from which the wind appears to be coming, so this bearing is
$$20.6° \text{ to 1 dp}$$

Question 8

$$y = (ax+3)\ln x$$

On the axis
$$(ax+3)\ln x = 0 \qquad (1)$$

Now
$$\frac{dy}{dx} = (ax+3)\frac{1}{x} + \ln x(a)$$

The normal to the curve at $y = 0$ is parallel to $5y + x = 2$, which is
$$y = \frac{1}{5}(-x+2)$$

which has gradient $-\frac{1}{5}$

So at $y = 0$, the curve has gradient 5, so
$$\frac{dy}{dx} = (ax+3)\frac{1}{x} + \ln x(a) = 5 \qquad (2)$$

From (1), letting
$$\ln x = 0 \Rightarrow x = 1$$

Into (2)
$$(a+3) = 5 \Rightarrow a = 2$$

From (1), letting
$$(ax+3) = 0 \Rightarrow a = -\frac{3}{x}$$

But since $x > 0$, this leads to a negative value for a, whereas the question states that a is positive

Question 9

(a) *Taking note of the general term for the expansion of*
$$(a+b)^n$$
being of the form
$${}^nC_r a^{n-r} b^r$$
we can go straight to the required term without doing the full expansion. We need to realize that the x-terms have to cancel out completely

Term we require is
$$x^{15}\frac{(18)(17)(16)}{3!}\left(-\frac{1}{2x^5}\right)^3$$
$$= \frac{4896}{6}\left(-\frac{1}{8}\right)$$
$$= -102$$

(b)

$$(1+kx)^n \quad n \geq 3$$

Require situation where

$$\frac{(n)(n-1)}{2!}(k)^2 = \frac{n(n-1)(n-2)}{3!}(k)^3$$

$$k^2 = \frac{(n-2)k^3}{3}$$

$$3 = (n-2)k$$

$$k = \frac{3}{n-2}$$

Question 10

(i) Since $\triangle ABC$ is isosceles

$$\angle ACB = \pi - 1.4$$

Area of $\triangle ABC$

$$= \frac{1}{2} \times 20 \times 20 \times \sin(\pi - 1.4)$$

$$= 197.08994.... = 197.1cm^2 \text{ to 1 dp}$$

(Remember to use your calculator in RADIAN mode)

Area of sector of circle ACD

$$= \pi(20)^2 \times \frac{0.7}{2\pi} = 140$$

So area of shaded area

$$= 197.1 - 140 = 57.1cm^2 \text{ to 1 dp}$$

Could also have found the area of sector by using the formula

$$Area = \frac{r^2\theta}{2}$$

(which is applicable as long as θ is in radians)

(ii) length of arc

$$= r\theta$$
$$= 20 \times 0.7$$
$$= 14$$

to calculate AB

$$\frac{20}{\sin 0.7} = \frac{AB}{\sin(\pi - 1.4)}$$

$$AB = \frac{20\sin(\pi - 1.4)}{\sin 0.7}$$
$$= 30.59368...$$

so

$$BD = 30.59368... - 20$$
$$= 10.59368...$$
$$= 10.6 \text{ to 3 sig figs}$$

total perimeter

$$= 14 + 10.6 + 20$$

$$= 44.6cm$$

Question 11

(i)

$$y = -\int \frac{a}{x^3}\, dx$$

$$y = -a\frac{x^{-2}}{-2} + c$$

$$y = \frac{a}{2}\frac{1}{x^2} + c$$

When $y = 3.5,\ x = 2$

$$3.5 = \frac{a}{8} + c \qquad (3)$$

When $y = 1.4,\ x = 5$

$$1.4 = \frac{a}{50} + c \qquad (4)$$

(3) - (4)

$$2.1 = \frac{a}{8} - \frac{a}{50}$$
$$\frac{50a}{400} - \frac{8a}{400} = 2.1$$
$$\frac{42a}{400} = 2.1$$
$$a = \frac{840}{42}$$
$$a = 20$$

Equation becomes

$$y = \frac{10}{x^2} + c$$

When $y = 3.5,\ x = 2$

$$3.5 = \frac{10}{4} + c$$

$$c = 3.5 - 2.5 = 1$$

so

$$y = \frac{10}{x^2} + 1$$

(ii) Using integral from above *(and ignoring c because we are dealing with a definite integral)*

$$\left[\frac{10}{x^2}\right]_2^p = \left[\frac{10}{x^2}\right]_p^5$$

$$\left(\frac{10}{p^2} - \frac{10}{2^2}\right) = \left(\frac{10}{5^2} - \frac{10}{p^2}\right)$$

$$\frac{10}{p^2} - 2.5 = \frac{10}{25} - \frac{10}{p^2}$$

$$10 - 2.5p^2 = \frac{10p^2}{25} - 10$$

$$250 - 62.5p^2 = 10p^2 - 250$$

$$72.5p^2 = 500$$

$$p^2 = \frac{500}{72.5}$$
$$= 6.896...$$

so

$$p = 2.6261.. = 2.63 \text{ to 3 sig figs}$$

Question 12 - OPTION 1

(a) (i)

$$\frac{12!}{4!8!}$$
$$= 495$$

(ii) Possibilities

Trig and Algebra: only 7 qns so not applicable

Trig and Calculus: 8 qns so only 1 selection possible

Algebra and Calculus: 9 qns, so no of possible selections

$$= \frac{9!}{1!8!} = 9$$

So possible number of selections

$$= 1 + 9 = 10$$

(b) (i)

$$\frac{8!}{3!}$$
$$= 6720$$

(ii)

$$\frac{6720}{8} = 840$$

(iii)

$$6720 \times \frac{5}{8} = 4200$$

Question 12 - OPTION 2

(i) Draw graph. The vertical values are

2.26...
2.96...
3.62...
4.30...
4.97...

(ii)

$$y = Ab^x$$
$$\lg y = \lg(Ab^x)$$
$$\lg y = \lg A + \lg b^x$$
$$\lg y = \lg A + x \lg b$$

Using data from graph, intersection of vertical axis at 1.6 gives

$$\lg A = 1.6$$
$$A = 4.9530... = 5.0 \text{ to 1 dp}$$

Gradient of graph

$$= \frac{1.7}{5} = 0.34$$
$$\lg b = 0.34$$
$$b = 1.4049..$$
$$= 1.4 \text{ to 1 dp}$$

(iii)

$$y = 2^x$$

so

$$\ln y = x \ln 2$$

and draw graph of this Estimated value in region of

$$x = 4.4$$

i.e. intersection of two graphs

Paper 2 - 0606/02, May/June 2004

Question 1

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 4 \\ k \end{bmatrix} - \begin{bmatrix} 1 \\ -7 \end{bmatrix}$$

$$0.8 = k - (-7)$$

$$k = -6.2$$

Question 2

$$10\cos\left(\frac{x+1}{2}\right) = 3$$

$$\cos\left(\frac{x+1}{2}\right) = \frac{3}{10}$$

$$\frac{x+1}{2} = 1.2661..$$

$$x = 2(1.2661..) - 1$$

$$= 1.5322..... = 1.53 \text{ to 3 sig figs}$$

Question 3

(i) The number of students who are both over 180cm tall and are vegetarian is not zero

(ii) All students over 180cm tall are non-cyclists

(iii)

$$(B \cap C) \cap A' \neq \emptyset$$

Question 4

$$(1 + \sec\theta)(cosec\theta - \cot\theta)$$

$$\left(1 + \frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)$$

$$\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + \frac{1}{\cos\theta\sin\theta} - \frac{\cos\theta}{\cos\theta\sin\theta}$$

The first and last term cancel. Adjusting the second term gives

$$\frac{-\cos^2\theta}{\sin\theta\cos\theta} + \frac{1}{\cos\theta\sin\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

Question 5

From the quadratic

$$c + d = \sqrt{20}$$

$$cd = 2$$

Now

$$\frac{1}{c} + \frac{1}{d} = \frac{c+d}{cd}$$

$$= \frac{\sqrt{20}}{2}$$

$$= \frac{\sqrt{4 \times 5}}{2} = \frac{\sqrt{4}\sqrt{5}}{2} = \sqrt{5}$$

Question 6

(a)

$$2x^2 > 3x + 14$$

$$2x^2 - 3x - 14 > 0$$

$$(2x-7)(x+2) > 0$$

so

$$2x - 7 = 0 \Rightarrow x = \frac{7}{2}$$

$$x + 2 = 0 \Rightarrow x = -2$$

These are the points where the sign of the function can change

	$x < -2$	$-2 < x < \frac{7}{2}$	$\frac{7}{2} < x$
2x-7	-	-	+
x+2	-	+	+
(2x-7)(x+2)	+	-	+

So required ranges are

$$x < -2 \quad \text{or} \quad x > \frac{7}{2}$$

(b) The curve is

$$x^2 + 4y = 20$$

$$\rightarrow 4y = -x^2 + 20$$

$$\rightarrow y = -\frac{1}{4}x^2 + 5$$

$$\frac{dy}{dx} = -\frac{1}{2}x$$

The given line is

$$y + kx = 8$$

$$y = -kx + 8$$

Comparing gradients

$$-\frac{1}{2}x = -k$$

So required values of k are given by

$$k = \frac{1}{2}x$$

Question 7

(i)

$$fg(x) = e^{2x-3}$$

so

$$fg(x) = 7$$
$$e^{2x-3} = 7$$
$$2x - 3 = \ln 7$$
$$2x = \ln 7 + 3$$
$$x = \frac{\ln 7 + 3}{2}$$
$$x = 2.4729...$$
$$= 2.47 \text{ to 3 sig figs}$$

(ii)

$$h = gf(x) = 2e^x - 3$$

Range > -3
the range relates to the range of relevant y-coordinates

(iii) Let

$$y = 2e^x - 3$$
$$2e^x = y + 3$$
$$e^x = \frac{y+3}{2}$$
$$x = \ln \frac{y+3}{2}$$

or

$$x = \ln(y+3) - \ln 2$$

and purely by changing the name of the variables

$$h^{-1}(x) = \ln \frac{x+3}{2}$$

Question 8

(i)

$$\log_3(2x+1) = 2 + \log_3(3x-11)$$
$$\log_3(2x+1) - \log_3(3x-11) = 2$$
$$\log_3 \frac{(2x+1)}{(3x-11)} = 2$$
$$\frac{2x+1}{3x-11} = 3^2$$
$$2x + 1 = 9(3x - 11)$$
$$2x + 1 = 27x - 99$$
$$25x = 100$$
$$x = 4$$

(ii)

$$\log_4 y + \log_2 y = 9$$
$$\frac{\log_2 y}{\log_2 4} + \log_2 y = 9$$
$$\frac{\log_2 y}{2} + \log_2 y = 9$$
$$\frac{3}{2}\log_2 y = 9$$
$$\log_2 y = \frac{18}{3} = 6$$
$$y = 2^6 = 64$$

Question 9

$$6 + 4x - x^2$$
$$= 6 - (-4x + x^2)$$
$$= 6 - \{(x-2)^2 - 4\}$$
$$= 10 - (x-2)^2$$

(i) From above, the expression becomes

$$y = 10 - (x-2)^2$$

this has a maximum value of

$$y = 10$$

when

$$(x - 2) = 0$$

$(x-2)^2$ *cannot be negative*

i.e. when

$$x = 2$$

Therefore this maximum is at

$$(2, 10)$$

(ii) When $x = 0,\ f = 6$
When $x = 5,\ f = 1$

And we know from above, that the curve has a maximum at $(2, 10)$, so Range of f is given by

$$1 \le \text{ Range } \le -10$$

(iii) No inverse because f is not one-to-one, i.e. when $x = 4, f = 6$ but also when $x = 0, f = 6$

Question 10

(i) Gradient of BC

$$= \frac{2-(-1)}{6-1} = \frac{3}{5}$$

which is also the gradient of AD

AD goes through $(-2, 4)$, so eqn of AD

$$= y - 4 = \frac{3}{5}(x - (-2))$$
$$y - 4 = \frac{3}{5}x + \frac{6}{5}$$
$$\underline{y = \frac{3}{5}x + \frac{26}{5}}$$

Gradient of AC

$$= \frac{4-2}{-2-6} = \frac{2}{-8} = -\frac{1}{4}$$

therefore gradient of CD = 4

so, since CD goes through (6,2), eqn. of CD

$$= y - 2 = 4(x - 6)$$
$$y - 2 = 4x - 24$$
$$\underline{y = 4x - 22}$$

(ii) Equating eqns for AD and CD

$$\frac{3}{5}x + \frac{26}{5} = 4x - 22$$
$$3x + 26 = 20x - 110$$
$$17x = 136$$
$$x = 8$$

and

$$y = 4(8) - 22 = 32 - 22 = 10$$

so coordinates of D are

$$(8, 10)$$

(iii) Finding the length of AC from its end coordinates

$$AC^2 = (8^2 + 2^2) = 64 + 4 = 68$$

and likewise

$$CD^2 = (2^2 + 8^2) = 64 + 4 = 68$$

so

$$AC = CD$$

therefore $\triangle ACD$ is isosceles

Question 11

(i)

$$y = (x+1)(2x-3)^{\frac{3}{2}}$$
$$\frac{dy}{dx} = (x+1)\frac{3}{2}(2x-3)^{\frac{1}{2}}(2) + (2x-3)^{\frac{3}{2}}(1)$$
$$= (2x-3)^{\frac{1}{2}}(3[x+1] + [2x-3])$$
$$= (2x-3)^{\frac{1}{2}}(3x+3+2x-3)$$
$$= (2x-3)^{\frac{1}{2}}(5x)$$

Therefore

$$k = 5$$

(ii)

$$y = ((6+p)+1)(2(6+p)-3)^{\frac{3}{2}}$$
$$= (6+p+1)(12+2p-3)^{\frac{3}{2}}$$
$$= (7+p)(9+2p)^{\frac{3}{2}}$$
$$= (7+p)(9^{\frac{3}{2}} + \frac{3}{2}9^{\frac{1}{2}}(2p)...)$$

ignoring higher order terms

$$y = (7+p)(27 + \frac{9}{2}(2p)...)$$
$$= (7+p)(27+9p.....)$$
$$= 189 + 63p + 27p + 9p^2$$
$$= 189 + 90p$$

(iii)

$$\int_2^6 x\sqrt{2x-3}\,dx$$
$$= \frac{1}{5}\int_2^6 5x\sqrt{2x-3}\,dx$$

by reference to (i), the integral becomes

$$y = \frac{1}{5}\left[(x+1)(2x-3)^{\frac{3}{2}}\right]_2^6$$
$$y = \frac{1}{5}\left\{\left((7)(9)^{\frac{3}{2}}\right) - \left((3)(1)^{\frac{3}{2}})\right)\right\}$$
$$= \frac{1}{5}(189-3)$$
$$= \frac{1}{5}(186)$$
$$= 37.2$$

Question 12-OPTION 1

(i)

$$v = 10\left(1 - e^{-\frac{1}{2}t}\right)$$
$$a = \frac{dv}{dt} = 10\left(\frac{1}{2}e^{-\frac{1}{2}t}\right)$$

Now, when $v = 8$, velocity equation becomes

$$8 = 10\left(1 - e^{-\frac{1}{2}t}\right)$$
$$1 - e^{-\frac{1}{2}t} = 0.8$$
$$e^{-\frac{1}{2}t} = 0.2$$

so

$$a = 10\left(\frac{1}{2}(0.2)\right) = 1ms^{-1}$$

(ii)

$$s = \int_0^6 10\left(1 - e^{-\frac{1}{2}t}\right)\,dt$$
$$= 10\left[t - \frac{e^{-\frac{1}{2}t}}{-\frac{1}{2}}\right]_0^6$$
$$= 10\left[t + 2e^{-\frac{1}{2}t}\right]_0^6$$
$$= 10\{(6 + 2e^{-3}) - (2)\}$$
$$= 10(4 + 2e^{-3})$$
$$= 40.9957.... = 41m \text{ to nearest meter}$$

(iii)

$$v = 10\left(1 - \frac{1}{e^{\frac{1}{2}t}}\right)$$

As $t \to \infty$, $\frac{1}{e^{\frac{1}{2}t}} \to 0$, so $v \to 10$

(iv) Draw graph. From origin originally, tending to v=10 at time progresses. Shape similar to that of a charging capacitor.

Question 12-OPTION 2

(i)

$$\frac{d}{d\theta}(\sec\theta) = \frac{d}{d\theta}(\cos\theta)^{-1}$$
$$= -(\cos\theta)^{-2}(-\sin\theta)$$
$$= \frac{\sin\theta}{\cos^2\theta}$$

(ii)

$$T = \frac{AX}{3} + \frac{XQ}{5}$$

Now

$$AX\cos\theta = 2$$
$$AX = \frac{2}{\cos\theta}$$

and

$$XQ = 10 - AX\sin\theta$$
$$= 10 - \frac{2}{\cos\theta}\sin\theta$$

inserting these expressions for AX and XQ into previous expression for T

$$T = \frac{2}{3\cos\theta} + \frac{1}{5}\left\{10 - \frac{2\sin\theta}{\cos\theta}\right\}$$
$$T = \frac{2\sec\theta}{3} + 2 - \frac{2\tan\theta}{5}$$

(iii)

$$\frac{dT}{d\theta} = \frac{2}{3}\sec\theta\tan\theta - \frac{2}{5}\sec^2\theta$$
$$= \frac{2}{3}\sin\theta\sec^2\theta - \frac{2}{5}\sec^2\theta$$
$$= \sec^2\theta\left(\frac{2}{3}\sin\theta - \frac{2}{5}\right)$$

at a stationary point

$$= \sec^2\theta\left(\frac{2}{3}\sin\theta - \frac{2}{5}\right) = 0$$

so

$$\frac{2}{3}\sin\theta - \frac{2}{5} = 0$$
$$\frac{2}{3}\sin\theta = \frac{2}{5}$$
$$\sin\theta = \frac{3}{5}$$
$$\Rightarrow \cos\theta = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$
$$= \sqrt{1 - \frac{9}{25}}$$
$$= \sqrt{\frac{16}{25}}$$
$$= \frac{4}{5}$$

Now

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4} = \frac{PX}{AP}$$

$AP = 2$

$$\frac{PX}{2} = \frac{3}{4}$$
$$PX = \frac{6}{4} = 1.5km$$